The Elitzur-Vaidman bomb test

São Paulo University
São Carlos Institute of Physics
Jonathas Queiroz Ribeiro Moraes - 9762661

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Abstract

The complex behavior of Quantum Physics for long puzzled scientists, who battled to understand and clarify its rather unique concepts. To achieve this, they tried to create simple ways to explain the phenomena that happen in the quantum world and didn't always have analogues to the classical understanding of Physics. In 1993, A. Elitzur and L. Vaidman published an article describing the possibility of making a quantum mechanical interaction-free measurement, an unprecedented claim that has no classical parallel. The bomb-testing problem was a way to visualize this experiment proposed by them, in which is possible to get information without interacting directly with the object of interest and also without prior knowledge of the state of the system. This article seeks to explore this problem and roll an experiment to better understand this method.

KEYWORDS: Quantum Physics; interaction-free measurement; bomb-tester.

1 Introduction

The works of Avshalom Elitzur and Lev Vaidman[1] showed that we actually can make a interaction-free measurement, which is a breakthrough affirmation in the field of quantum mechanics. This is so unique because there's no analogue to this behavior in classical mechanics, i.e., in a classical measurement, there needs to be a form of interaction with the object we're trying to acquire information from.

In classical mechanics, an object with an electric or magnetic moment can be detected indirectly via measuring the electromagnetic field it creates in its surroundings, with no need for a particle to pass through that field. However, via the Aharonov-Bohm effect[2], we can infer the existence of an object in a non-local way even when the object creates no electromagnetic field in a region, but a potential.

In simple means, it is possible to make a interaction-free non-local measurement, for example when we know that one ball is inside one of two boxes and simply by opening one box and finding that the ball isn't there assures us the location of the ball, without even interacting locally with it. And this is only possible because, in order to acquire information performing this interaction-free measurement, we need to have an information about the object prior to the measurement. But what Elitzur and Vaidman did was to prove that is possible to make the measurement even without any previous information.

2 Measuring without interaction

In their experiment, there was a Mach-Zehnder interferometer with beam-splitters of $\frac{1}{2}$ reflectivity. The light from the source reaches the first beam-splitter and is divided equally between two paths, A (upper) and B (lower). These paths have mirrors, which redirect the beams to the final beam-splitter. Finally, we have two detectors after the latter beam-splitter, and the interferometer is build in such a way that the beams interfere constructively in one of the detectors (D_1) and destructively in the other (D_2). This means that, when the two paths are free, we'll always detect light only at D_1. However, when somehow one path is blocked, it is possible to detect light in both detectors equally. Elitzur and Vaidman propose this experiment in a specific set of conditions, for a light source that only emits one photon at a time, in a controllable way.

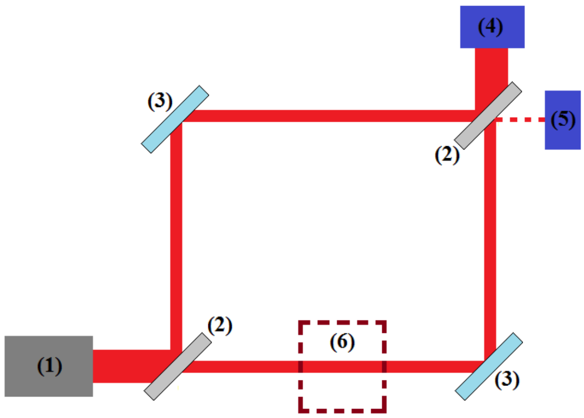

Figure 2.1: The laser (1) emits a beam, which is divided by the first of the beam-splitters (2). Both arms have a mirror (3) and when the beams encounter in the second beam-splitter, they interfere in a way that only the detector D_1 (4) sees light coming and D_2 (5) doesn't. However, when an object (6) is placed, blocking one of the arms, light may be detected on D_2 as well.
Font: Author.

The image above illustrates the Mach-Zehnder interferometer composition for this experiment.

For the case of single photon emissions, the outcomes of the interferometer are limited to those below:

i. no detector clicks: this happens when the photon interacts with the object put in the beam's path, thus, the photon never reaches any detector, with probability $\frac{1}{2}$;

ii. detector D_1 clicks: this happens both when the object is present, with a probability of $\frac{1}{4}$, and always happens when it is not present; therefore, the measurement hasn't succeeded and we can give it another try;

iii. detector D_2 clicks: this happens also with a probability of $\frac{1}{4}$, and this is what we're expecting – a measurement of the presence of an object without interacting with it, as we would do in the first case.

It's important to emphasize that we can assure that a detection on D_2 only happens because there is an object in the path of the photon, since we set the interferometer to a destructive interference in it.

Mathematically, describing the state of a photon moving right as $|1\rangle$ and up as $|2\rangle$, the operation for a beam-splitter is

$$\begin{aligned} |1\rangle &\xrightarrow{BS} \frac{1}{\sqrt{2}}(|1\rangle + i|2\rangle) \\ |2\rangle &\xrightarrow{BS} \frac{1}{\sqrt{2}}(|2\rangle + i|1\rangle) \end{aligned} \tag{2.1}$$

and for a mirror

$$\begin{aligned} |1\rangle &\xrightarrow{M} i|2\rangle \\ |2\rangle &\xrightarrow{M} i|1\rangle \end{aligned} . \tag{2.2}$$

Therefore, when the system doesn't have the object placed in~~t~~ any arm we have, as we should expect,

$$\begin{aligned}|1\rangle &\xrightarrow{BS} \frac{1}{\sqrt{2}}\left(|1\rangle + i|2\rangle\right) \xrightarrow{M} \frac{1}{\sqrt{2}}\left(i|2\rangle - |1\rangle\right) \\ &\xrightarrow{BS} \frac{1}{2}\left(i|2\rangle - |1\rangle\right) - \frac{1}{2}\left(|1\rangle + i|2\rangle\right) = -|1\rangle\end{aligned} \quad . \tag{2.3}$$

But if there is an object in the path, another state $|s\rangle$ can occur, when the photon is scattered, leading us to another result

$$\begin{aligned}|1\rangle &\xrightarrow{BS} \frac{1}{\sqrt{2}}\left(|1\rangle + i|2\rangle\right) \xrightarrow{M} \frac{1}{\sqrt{2}}\left(i|2\rangle + i|s\rangle\right) \\ &\xrightarrow{BS} \frac{1}{2}\left(i|2\rangle - |1\rangle\right) + \frac{i}{\sqrt{2}}|s\rangle\end{aligned} \tag{2.4}$$

which yields the probabilities as we discussed on the three different cases before

$$\begin{cases} |1\rangle, D_1 \text{ clicks,} & \text{with prob. } \frac{1}{4} \\ |2\rangle, D_2 \text{ clicks,} & \text{with prob. } \frac{1}{4} \\ |s\rangle, \text{ neither clicks,} & \text{with prob. } \frac{1}{2} \end{cases} . \tag{2.5}$$

3 Bomb-testing problem

Suppose now we have a stock of bombs, each with an extremely precise sensor that if it detects a single photon, the bomb explodes. But not all the sensors are working: we have some defective bombs, in which the photon doesn't interact with the sensor, passing through it, and the bomb doesn't detonate. Is is possible to verify which of the bombs are working and which are not?

Primarily, we want to detect working bombs without exploding them, in order to use them later. So just directing light on all sensors won't solve our problem. To be able to test these bombs properly in the way we want, we shall use the principles of quantum mechanics we discussed.

By placing the bomb's sensor in one of the arms of the interferometer, we can send individual photons until either the bomb explodes or D_2 detects a photon, in order to know the bomb we tested was good. If we put a working bomb in the interferometer, the photon has a $\frac{1}{2}$ chance to detonate the bomb. In the other case, it proceeds to the latter beam-splitter, where it has an equal probability to go either to detector D_1 or D_2. If it hits D_2, with probability $\frac{1}{4}$, the bomb was good and didn't explode. If the photon is detected in D_1, with probability also $\frac{1}{4}$, we shall send new photons, until after a sufficiently large number of tests we may affirm that the bomb we were testing was or wasn't good.

In the limit where the bomb isn't destroyed, we sent infinitely many photons that hit the detector D_1 until one photon finally hit D_2. This gives us a $\sum_{n=1}^{\infty} \frac{1}{4^n} = \frac{1}{3}$ probability of detecting a good bomb without destroying it.

If however we change the mirror reflectivity on the beam-splitter, it is possible to increase the probability of detecting the photon on D_2. We model the first beam-splitter to behave as

$$\begin{aligned}|1\rangle &\xrightarrow{BS_1} & a|1\rangle + ib|2\rangle \\ |2\rangle &\xrightarrow{BS_1} & a|2\rangle + ib|1\rangle\end{aligned} \tag{3.1}$$

and the second as

$$\begin{aligned}|1\rangle &\xrightarrow{BS_2} & b|1\rangle + ia|2\rangle \\ |2\rangle &\xrightarrow{BS_2} & b|2\rangle + ia|1\rangle\end{aligned} \quad , \tag{3.2}$$

where $a^2 + b^2 = 1$ and $a >> b$. With a similar treatment as the one we did before

$$\begin{aligned}|1\rangle &\xrightarrow{BS_1} a|1\rangle + ib|2\rangle \xrightarrow{M} ia|2\rangle + ib|a\rangle \\ &\xrightarrow{BS_2} ia\left[b|2\rangle + ia|1\rangle\right] + ib|a\rangle\end{aligned} \quad , \tag{3.3}$$

in which $|a\rangle$ represents the state where the photon is absorbed. This gives us the following probabilities for each case

$$\begin{cases} |1\rangle, D_1 \text{ clicks, no explosion} & \text{prob. } a^4 \\ |2\rangle, D_2 \text{ clicks, no explosion} & \text{prob. } a^2 b^2 \\ |s\rangle, \text{neither clicks, explosion} & \text{prob. } b^2 \end{cases} . \tag{3.4}$$

This final result gives us the the probability of detecting the photon on D_2 if the photon doesn't hit the bomb is a^2. This follows from the ratio between the probability of detecting D_2 and the probability of the bomb exploding – which you can verify too for the case when the reflectivity is $\frac{1}{2}$ $(a = b = \frac{1}{\sqrt{2}})$.

And since $a >> b$, a is close to 1 and we can be almost certain that we can detect good bombs without exploding them. We might add that, in this particular case, the probability of detecting the first photon at D_2 is small, so we might have various photons being sent (or the same many times) in order to determine the reliability of the bomb.

4 Experiment

In order to better understand this problem, we'll simulate a bomb-testing problem with a educational kit from Thor Labs[(a)].

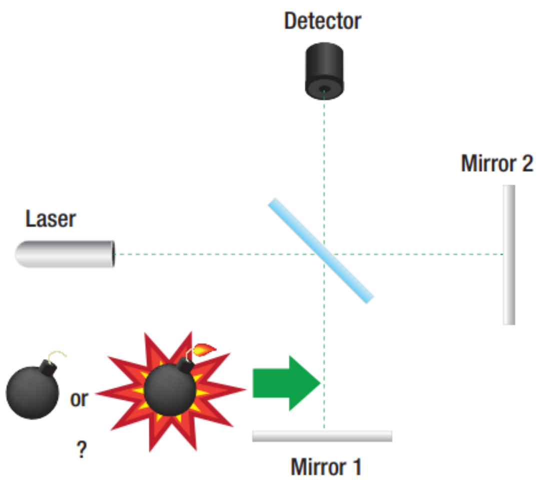

Figure 4.1: Thor Labs Bomb Tester Demonstration Kit diagram.
Font: Thor Labs.

This kit operates with a Michelson interferometer mount, different from the Mach-Zehnder proposed by Elitzur and Vaidman, but this doesn't affect the results – it only changes the methods to acquire data. Since we're only interested in detecting light on D_2, D_1 is dispensable. Therefore, the part of the beam that returns to the laser in a Michelson interferometer might be treated as D_1 and therefore is expendable to our measurements.

The image above illustrates the kit mount, and the following presents the experimental kit configuration.

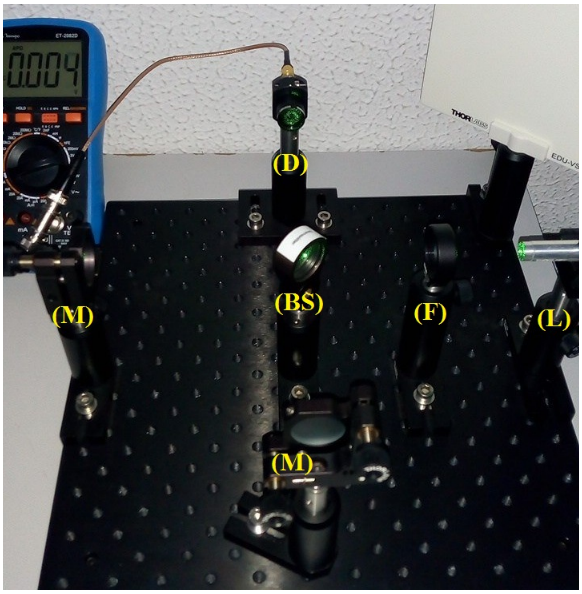

Figure 4.2: The laser (L) emits a beam, which is magnified via a lens (F) before it hits the beam-splitter (BS). Both arms of the interferometer have a mirror (M) and when the beams encounter each other at the beam-splitter, they interfere, following then to the detector (D) and back to the laser as well.
Font: Author.

One interesting thing to point is that in the original problem we discussed the sending of single photons at a time, which is hard to acquire and surely not suitable to turn compact for an educational design. Since our light source is a continuous laser sending many photons, we might adapt our measurements to our mount.

[(a)]Thor Labs - Bomb Tester Demonstration Kit

Instead of a single-photon detector, we use a photodiode detector, that measures light intensity. This means we're not reading single-photons, but the probabilities of detecting a photon in a given path in the interferometer, based on the total and measured intensities.

Therefore, we'll set the constructive interference to coincide with the detector opening, regulating the amount of light that reaches it by varying the aperture of its iris. By blocking the light of one of the paths, simulating the presence of a working bomb, the light intensity at the detector might read 25% of the total intensity, since the light was split in two at the beam-splitter, one half hitting the bomb and the other half being reflected and reaching the beam-splitter again, dividing itself equally towards the laser and the detector. This simulates the probability of 25% of detecting a photon at the detector when a live bomb, and not a dud bomb, is placed in one of the arms of the interferometer, as we discussed before.

The following image illustrates the interference pattern that we might find after the laser travels through the interferometer, so that we'll try to align its central ring to the detector's center.

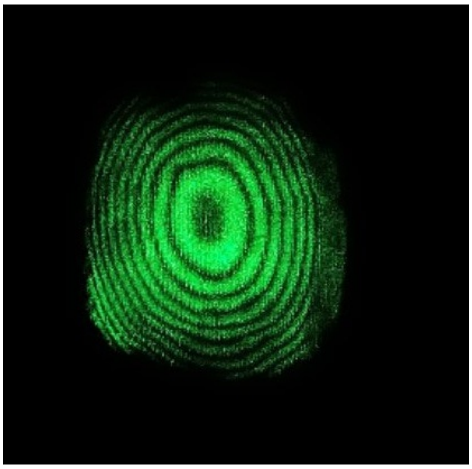

Figure 4.3: Interference pattern seen at a screen further away than the position of the detector.
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5 Results and Discussion

Measurements were made after various trials to excel the alignment of the beam an the optical components of the interferometer. It was quite difficult to center the constructive interference properly at the center of the detector. However, this was accomplished and 11 measurements were made, in order to reduce the error in the results. We obtained the following results, shown on the graphic below.

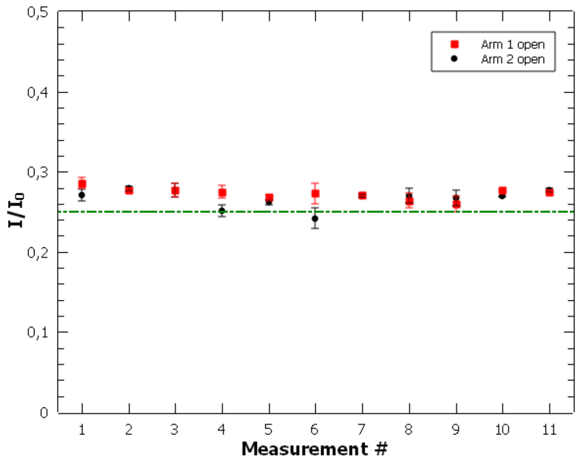

Figure 5.1: Percentage of total intensity achieved by blocking each of the arms in different configurations of distance and aperture. The dotted green line represents the desired 25% value.
Font: Author.

Three intensity reads were made each time: the total intensity of the construction interference at the beam center; and the intensities when each of the arms of the interferometer were blocked, leaving the other open. Red represents arm 1 being open and black represents arm 2 being open. The graphic's y axis represents the ratio of intensity relative to the total intensity of the beam. Therefore, we would expect that the values we measured should lie close to the green line represented in the graphic, whit the 25% intensity we're looking for.

The data set for each arm gives a mean intensity percentage of $27.3 \pm 0.6\%$ for arm 1 and $26.7 \pm 0.6\%$ for arm 2. These results are a bit off of the 25% we expected, but the discrepancies may be explained by losses at the optical apparatus or caused by dust, measurement inaccuracies and also the light noise that we couldn't determine since it was difficult to position the destructive interference at the center of the detector.

Nevertheless, these values are very close to what the theory predicts and we can say with no exaggeration that the Thor Labs Bomb Tester Demonstration Kit functions as expected, given the approximations we made for such a simple and didactic mount.

6 Conclusion

The results obtained by this experiment corroborate the results predicted by Avshalom Elitzur and Lev Vaidman, whose works laid important foundations for the field of Quantum Physics, specially the concept of making an interaction-free measurement. Even if the first assumptions may be simple and the bomb-testing problem may be a didactic way of explaining it, the consequences and further generalizations of this type of measurement were crucial for the years of research that would come, such as the selection of atoms in a excited metastable state without interacting with them, for example.

Our experiment shows with good precision, for a simple approach to this complex problem, that is indeed possible to prove and see the veracity of quantum superposition and interaction-free measurements, that wouldn't be possible in any classical experiment.

References

[1] A. C. Elitzur and L. Vaidman, "Quantum mechanical interaction-free measurements", Foundation of Physics **23**, 987–997 (1993).

[2] Y. Aharonov and D. Bohm, "Significance of electromagnetic potentials in the quantum theory", Physical Review **115**, 485–491 (1959).