# SU(3) flavour symmetry

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Mainly from "Modern Particle Physics", by Mark Thomson

Extension of flavour SU(2) to include the *s* quark

SU(2)

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix} \quad \begin{matrix} Q_q = +2/3 \\ Q_q = -1/3 \end{matrix}$$

SU(3)

$$m_{u,d} \approx 5\,\mathrm{MeV} \qquad m_c \approx 1.2\,\mathrm{GeV} \qquad m_t \approx 170\,\mathrm{GeV}$$

$$m_s \approx 100\,\mathrm{MeV} \qquad m_b \approx 4.8\,\mathrm{GeV}$$

$$m_{u,d,s} \ll m_{c,b,t}$$

The mass of the *s* quark is smaller than the masses of the *c*, *b* and *t* quarks and the typical binding energies of hadrons. But $m_s > m_{u,d}$ .

**Flavour SU(3) symmetry is less good than flavour SU(2)**

# Flavour SU(3)

Extend quark flavour rotations (3x3 matrices)

$$\begin{pmatrix} \mathrm{u}' \\ \mathrm{d}' \\ \mathrm{s}' \end{pmatrix} = \hat{U} \begin{pmatrix} \mathrm{u} \\ \mathrm{d} \\ \mathrm{s} \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} \mathrm{u} \\ \mathrm{d} \\ \mathrm{s} \end{pmatrix}$$

- 9x2=18 real parameters
- but unitarity imposes 9 constraints
- out of the 9 matrices, one is the identity: we need to build 8 generators.

fundamental (defining) representation of SU(3)

$$\hat{U} = e^{i\alpha\cdot\hat{\mathbf{T}}} \qquad \hat{\mathbf{T}} = \frac{1}{2}\lambda \qquad \mathrm{u} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathrm{d} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \mathrm{s} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

But $SU(3) \supset SU(2)$

Pauli matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

diagonal generator

Third component of isospin is now

$$T_3 = \frac{\lambda_3}{2}$$

Third component of isospin is now

$$T_3 = \frac{\lambda_3}{2}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

s quark doesn't carry isospin

$$\hat{T}_3 \mathrm{u} = +\tfrac{1}{2}\mathrm{u}, \quad \hat{T}_3 \mathrm{d} = -\tfrac{1}{2}\mathrm{d} \quad \text{and} \quad \hat{T}_3 \mathrm{s} = 0$$

Isospin ladder operators remain the same: $T_\pm = T_1 \pm iT_2$

There should be two additional embedded SU(2) for $u \leftrightarrow s$ and $d \leftrightarrow s$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \text{and} \quad \lambda_X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \text{and} \quad \lambda_Y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

but $\lambda_3 = \lambda_X - \lambda_Y$

We eliminate one of the linearly dependent matrices:

$$\lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

This combination leaves the light quarks unchanged.

2nd diagonal generator: the group is "rank two"
[SU(n) has n-1 diagonal generators]

# SU(3) generators: **Gell-Mann matrices**

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

SU(3) algebra

$$[\lambda_i, \lambda_j] = 2i\, f_{ijk} \lambda_k \qquad\qquad [T_i, T_j] = i\, f_{ijk} T_k$$

Antisymmetric $f_{ijk}$

Structure constants of SU(3)

# Structure constants of SU(3)

$$[\lambda_i, \lambda_j] = 2i\, f_{ijk}\lambda_k$$

$$f^{123} = 1$$
$$f^{147} = -f^{156} = f^{246} = f^{257} = f^{345} = -f^{367} = \frac{1}{2}$$
$$f^{458} = f^{678} = \frac{\sqrt{3}}{2},$$

(the others are zero)

# Quark SU(3) quantum numbers

$$\hat{T}^2 = \sum_{i=1}^{8} \hat{T}_i^2 = \frac{1}{4}\sum_{i=1}^{8} \lambda_i^2 = \frac{4}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

quadratic Casimir

$T_3 = \frac{1}{2}\lambda_3$ and $T_8 = \frac{1}{2}\lambda_8$ commute

Two quantum numbers: third component of isospin and *hypercharge* (eigenvalues of diag. gen)

$$\hat{T}_3 = \tfrac{1}{2}\lambda_3 \quad \text{and} \quad \hat{Y} = \frac{1}{\sqrt{3}}\lambda_8.$$

$$\begin{aligned} \hat{T}_3 \mathrm{u} &= +\tfrac{1}{2}\mathrm{u} \quad \text{and} \quad \hat{Y}\mathrm{u} = +\tfrac{1}{3}\mathrm{u}, \\ \hat{T}_3 \mathrm{d} &= -\tfrac{1}{2}\mathrm{d} \quad \text{and} \quad \hat{Y}\mathrm{d} = +\tfrac{1}{3}\mathrm{d}, \\ \hat{T}_3 \mathrm{s} &= 0 \quad \text{and} \quad \hat{Y}\mathrm{s} = -\tfrac{2}{3}\mathrm{s}. \end{aligned}$$

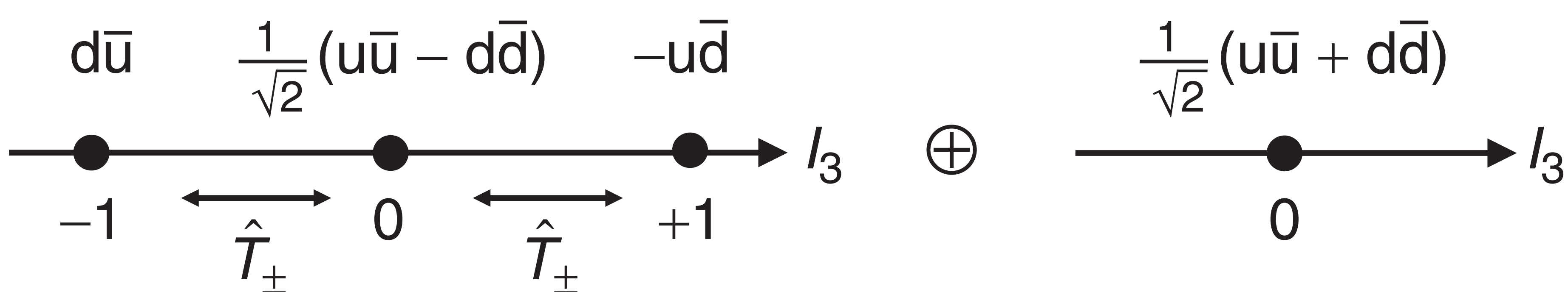
$$I_3 = \frac{1}{2}(n_u - n_d) \qquad Y = \frac{1}{3}(n_u + n_d - 2n_s)$$

additive quantum numbers

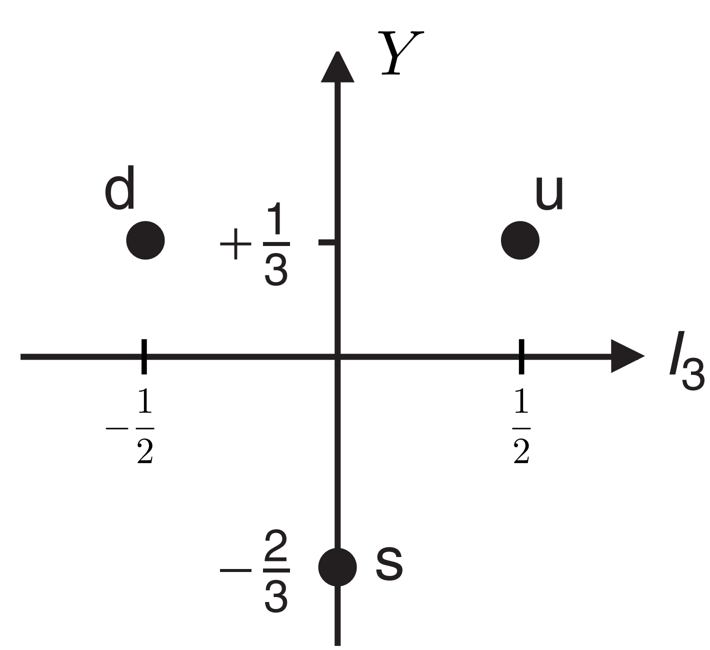
# SU(2) vs SU(3)

SU(2) is rank one: "one dimensional"


Mesonic states




SU(3) is rank two: bi-dimensional graphic representation

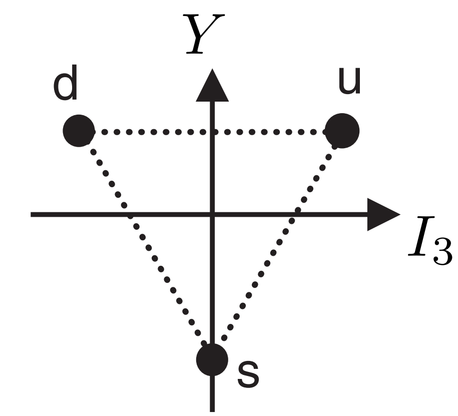


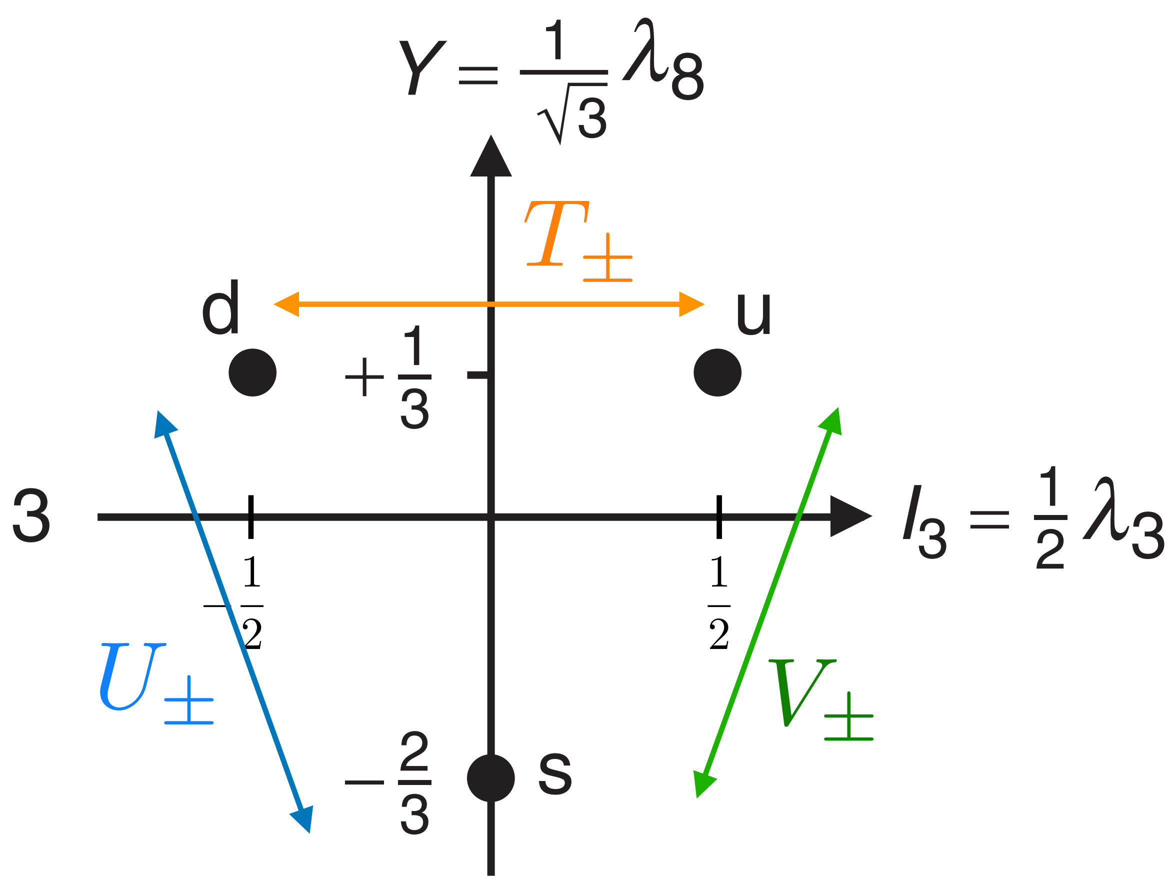
# **Quarks:** SU(3) quantum numbers

SU(3) is rank two:
"two dimensional"

$$\hat{T}_3 \mathrm{u} = +\tfrac{1}{2}\mathrm{u} \quad \text{and} \quad \hat{Y}\mathrm{u} = +\tfrac{1}{3}\mathrm{u},$$
$$\hat{T}_3 \mathrm{d} = -\tfrac{1}{2}\mathrm{d} \quad \text{and} \quad \hat{Y}\mathrm{d} = +\tfrac{1}{3}\mathrm{d},$$
$$\hat{T}_3 \mathrm{s} = 0 \quad \text{and} \quad \hat{Y}\mathrm{s} = -\tfrac{2}{3}\mathrm{s}.$$

fundamental 3 representation of SU(3)



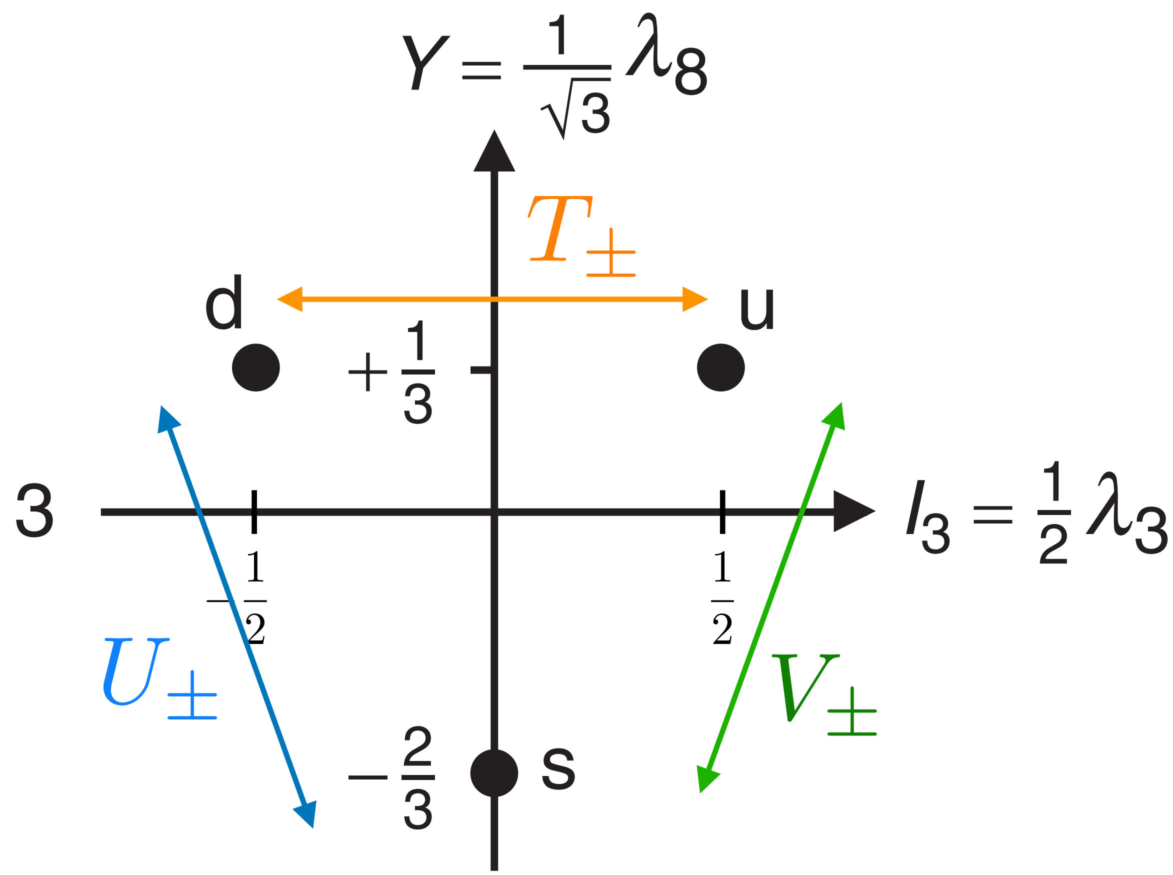

Three sets of ladders operators

$$\hat{T}_\pm = \tfrac{1}{2}(\lambda_1 \pm i\lambda_2),$$
$$\hat{V}_\pm = \tfrac{1}{2}(\lambda_4 \pm i\lambda_5),$$
$$\hat{U}_\pm = \tfrac{1}{2}(\lambda_6 \pm i\lambda_7),$$

$$\hat{V}_+\mathrm{s} = +\mathrm{u}, \quad \hat{V}_-\mathrm{u} = +\mathrm{s}, \quad \hat{U}_+\mathrm{s} = +\mathrm{d}, \quad \hat{U}_-\mathrm{d} = +\mathrm{s}, \quad \hat{T}_+\mathrm{d} = +\mathrm{u} \;\; \text{and} \;\; \hat{T}_-\mathrm{u} = +\mathrm{d}$$

$$\hat{V}_+ u = \hat{V}_- s = \hat{U}_+ d = \hat{T}_+ u \cdots = 0$$

# **Quarks:** SU(3) quantum numbers




Three sets of ladders operators

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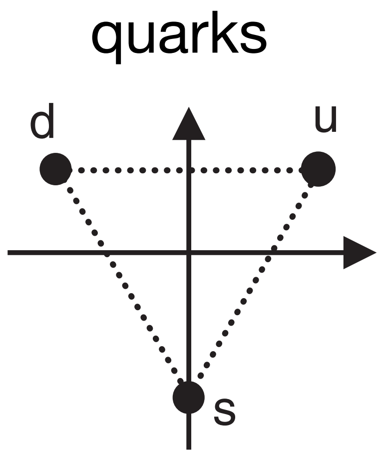
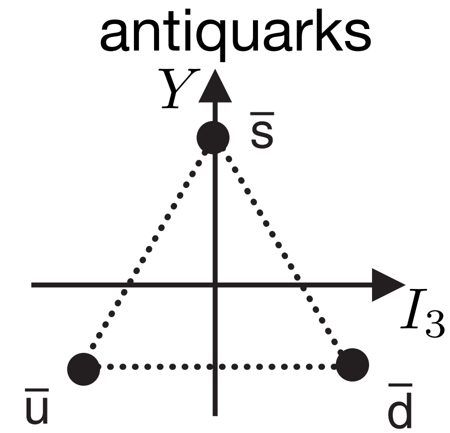
$$\hat{V}_+\mathrm{s} = +\mathrm{u}, \quad \hat{V}_-\mathrm{u} = +\mathrm{s}, \quad \hat{U}_+\mathrm{s} = +\mathrm{d}, \quad \hat{U}_-\mathrm{d} = +\mathrm{s}, \quad \hat{T}_+\mathrm{d} = +\mathrm{u} \;\text{ and }\; \hat{T}_-\mathrm{u} = +\mathrm{d}$$

In matrix form we have, for example:

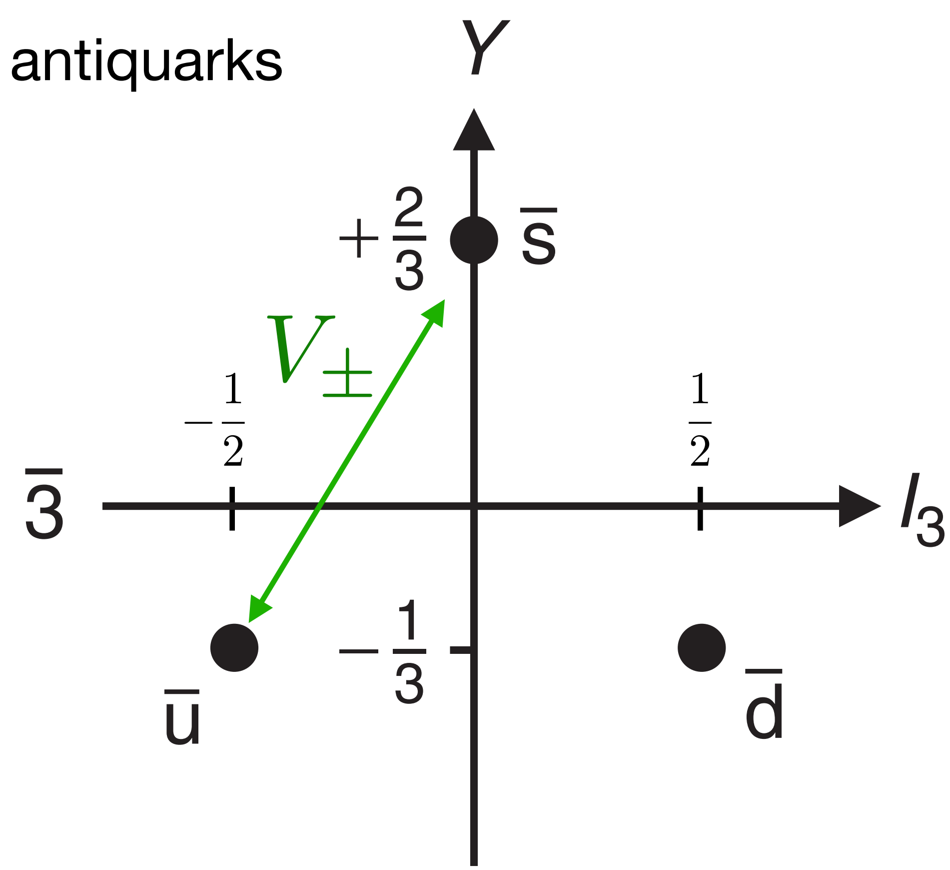
$$V_+ = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad V_+\, s = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = u$$

# **Antiquarks:** opposite quantum numbers




3* representation of SU(3) [in SU(2) the fundamental is a "real representation"]
also called conjugate representation ( also $\bar{3}$ )




$$u \equiv |1\rangle,\ d \equiv |2\rangle,\ s \equiv |3\rangle,$$

$$\bar{s} = ud - du = |1\rangle|2\rangle - |2\rangle|1\rangle$$

$$\bar{u} = |2\rangle|3\rangle - |3\rangle|2\rangle$$

$$\bar{d} = |1\rangle|3\rangle - |3\rangle|1\rangle$$

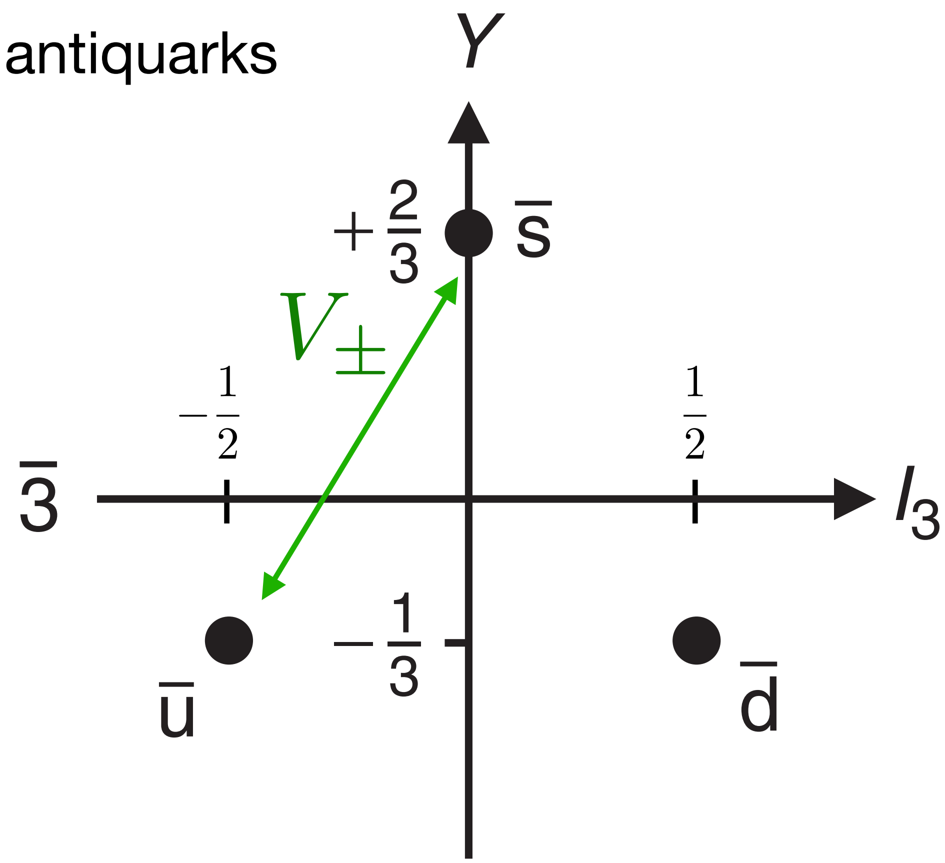
$$|\bar{i}\rangle = \epsilon^{ijk}|j\rangle|k\rangle$$

only way to get the correct quantum numbers!

eigenvalues of diagonal generators do not appear in pairs +m, -m, +Y, -Y, so the representation cannot be real

$$\hat{V}_+\bar{\mathrm{u}} = -\bar{\mathrm{s}},\quad \hat{V}_-\bar{\mathrm{s}} = -\bar{\mathrm{u}},\quad \hat{U}_+\bar{\mathrm{d}} = -\bar{\mathrm{s}},\quad \hat{U}_-\bar{\mathrm{s}} = -\bar{\mathrm{d}},\quad \hat{T}_+\bar{\mathrm{u}} = -\bar{\mathrm{d}} \ \text{and}\ \hat{T}_-\bar{\mathrm{d}} = -\bar{\mathrm{u}}.$$

# **Antiquarks:** opposite quantum numbers




$$u \equiv |1\rangle, \; d \equiv |2\rangle, \; s \equiv |3\rangle,$$

$$\bar{s} = ud - du = |1\rangle|2\rangle - |2\rangle|1\rangle$$
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$$|\bar{i}\rangle = \epsilon^{ijk}|j\rangle|k\rangle$$

$$\hat{V}_+\bar{\mathrm{u}} = -\bar{\mathrm{s}}$$

$$\hat{V}_+\bar{u} = (V_+ \otimes 1 + 1 \otimes V_+)(ds - sd) = du - ud = -\bar{s}$$

$$(\text{using} \;\; \hat{V}_+ s = u, \; \hat{V}_+ d = 0)$$

# Quantum number of hadronic states

$$I_3 = \frac{1}{2}(n_u - n_d) \qquad Y = \frac{1}{3}(n_u + n_d - 2n_s)$$

(flipping signs for antiquarks)

**próton**

$$(uud) \longrightarrow I_3 = \frac{1}{2}(2-1) = \frac{1}{2}; \qquad Y = \frac{1}{3}(2+1-0) = 1$$

**nêutron**

$$(udd) \longrightarrow I_3 = \frac{1}{2}(1-2) = -\frac{1}{2}; \qquad Y = \frac{1}{3}(1+2) = 1$$

$\pi^+$

$$(u\bar{d}) \longrightarrow I_3 = \frac{1}{2}(1+1) = +1; \qquad Y = \frac{1}{3}(1-1) = 0$$

$K^+$

$$(u\bar{s}) \longrightarrow I_3 = \frac{1}{2}(1-0) = +\frac{1}{2}; \qquad Y = \frac{1}{3}(1+2) = 1$$

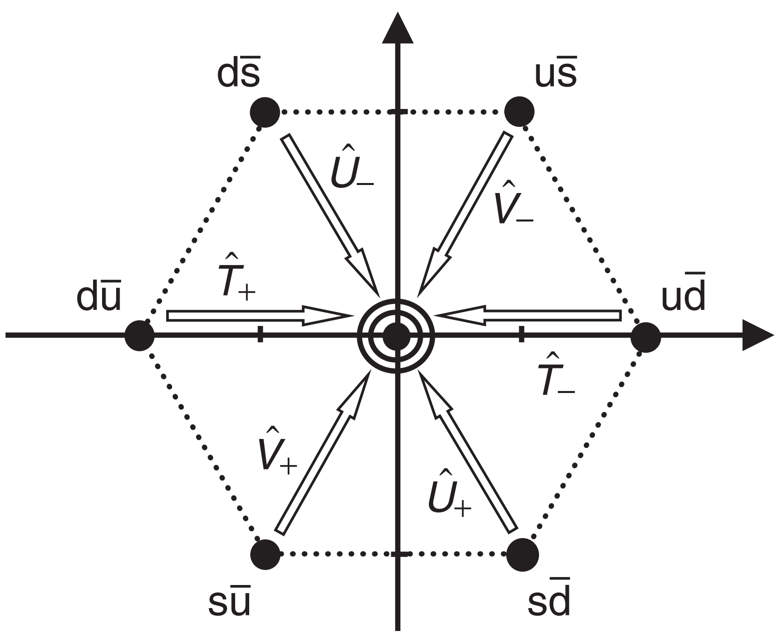
# Mesonic states $(q\bar{q})$

$I_3$ and $Y$ are additive quantum numbers.

Extreme states are easiest to obtain. From them, use ladder operators.

$$K^+ \quad (u\bar{s}) \longrightarrow I_3 = \frac{1}{2}(1-0) = +\frac{1}{2}; \qquad Y = \frac{1}{3}(1+2) = 1$$

$$K^- \quad (d\bar{s}) \longrightarrow I_3 = -\frac{1}{2}; \qquad Y = 1$$




Neutral states are a combination of the three $q\bar{q}$

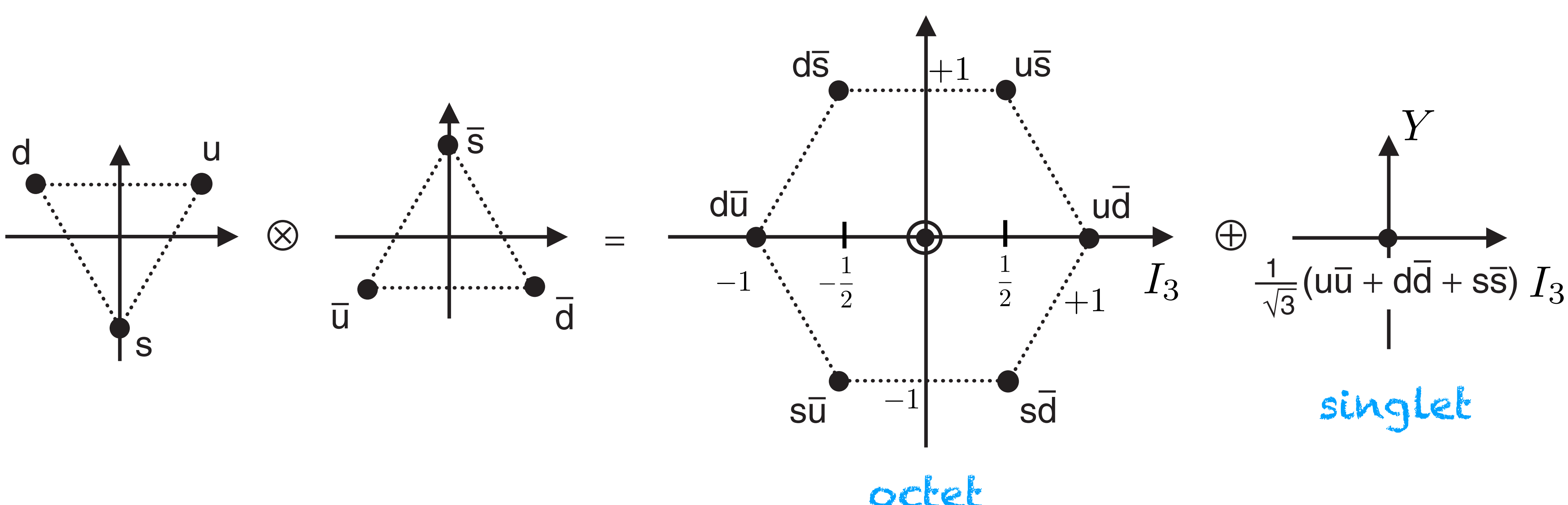
$$T_+|\mathrm{d\bar{u}}\rangle = |\mathrm{u\bar{u}}\rangle - |\mathrm{d\bar{d}}\rangle \quad \text{and} \quad T_-|\mathrm{u\bar{d}}\rangle = |\mathrm{d\bar{d}}\rangle - |\mathrm{u\bar{u}}\rangle,$$

$$V_+|\mathrm{s\bar{u}}\rangle = |\mathrm{u\bar{u}}\rangle - |\mathrm{s\bar{s}}\rangle \quad \text{and} \quad V_-|\mathrm{u\bar{s}}\rangle = |\mathrm{s\bar{s}}\rangle - |\mathrm{u\bar{u}}\rangle,$$

$$U_+|\mathrm{s\bar{d}}\rangle = |\mathrm{d\bar{d}}\rangle - |\mathrm{s\bar{s}}\rangle \quad \text{and} \quad U_-|\mathrm{d\bar{s}}\rangle = |\mathrm{s\bar{s}}\rangle - |\mathrm{d\bar{d}}\rangle.$$

only two are linearly independent:

6+2 = octet of states




singlet state

$$|\psi_S\rangle = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

$$(T_\pm/V_\pm/U_\pm)|\psi_S\rangle = 0$$

# Comment on the singlet state



singlet

$$|\psi_S\rangle = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$
$$= \frac{1}{\sqrt{3}}|i\rangle|\bar{i}\rangle$$
$$= \frac{1}{\sqrt{3}}|i\rangle\left(\epsilon^{ijk}|j\rangle|k\rangle\right)$$
$$= \frac{1}{\sqrt{3}}\epsilon^{ijk}|i\rangle|j\rangle|k\rangle$$

The singlet state is the totally antisymmetric combination of states

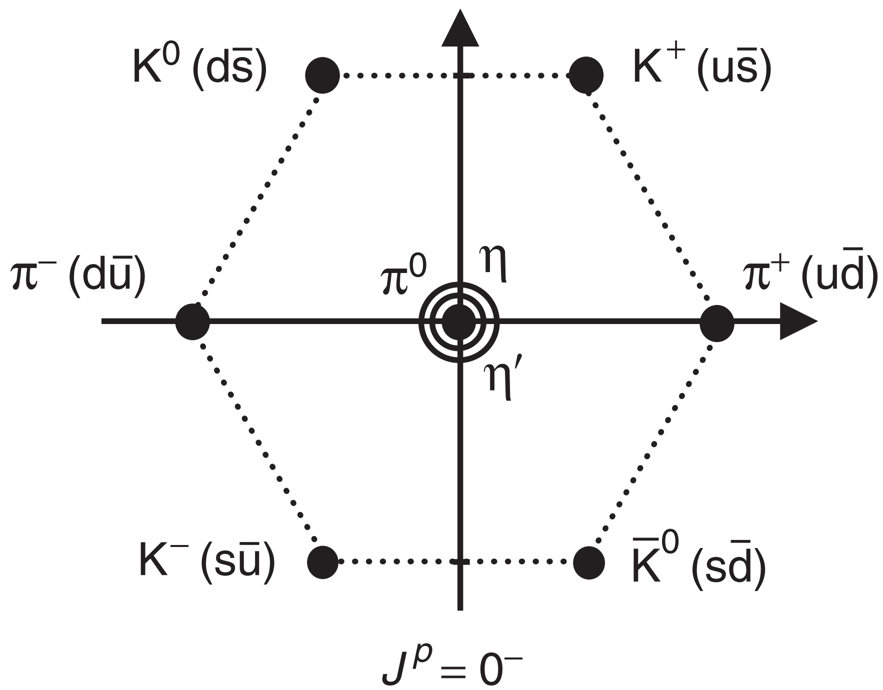
antiquarks

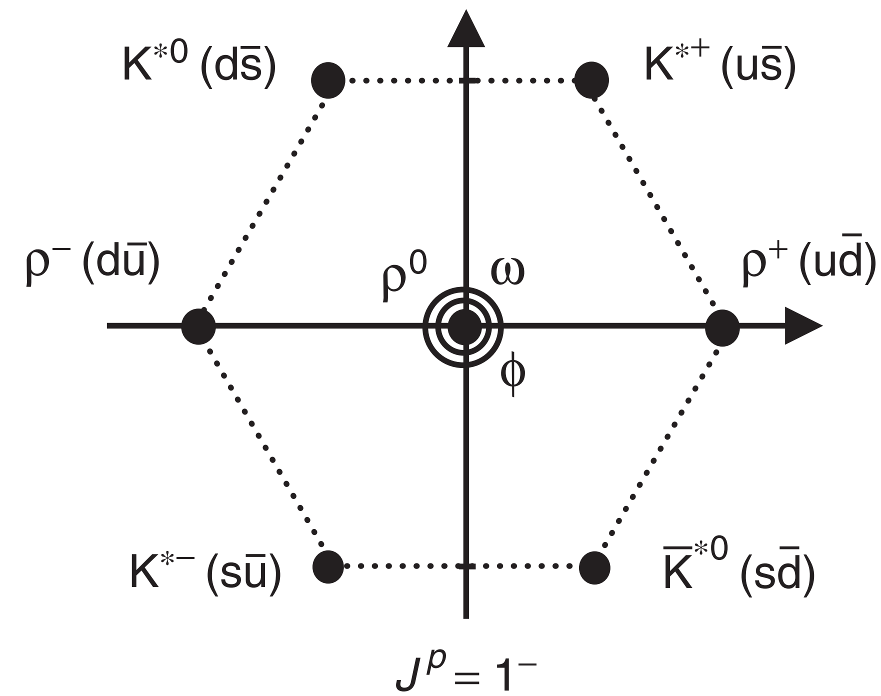
$$u \equiv |1\rangle,\ d \equiv |2\rangle,\ s \equiv |3\rangle,$$

$$\bar{s} = ud - du = |1\rangle|2\rangle - |2\rangle|1\rangle$$
$$\bar{u} = |2\rangle|3\rangle - |3\rangle|2\rangle$$
$$\bar{d} = |1\rangle|3\rangle - |3\rangle|1\rangle$$

$$|\bar{i}\rangle = \epsilon^{ijk}|j\rangle|k\rangle$$

# Mesonic states $(\ell = 0,\ s = 0 \text{ and } \ell = 0,\ s = 1)$




$$|\pi^0\rangle = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$

$$|\eta\rangle = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

octet I=0, Y=0 states

$$|\eta'\rangle \approx \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

singlet I=0, Y=0 states

$$|\rho^0\rangle = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}),$$

$$|\omega\rangle \approx \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}),$$

$$|\phi\rangle \approx s\bar{s}.$$

for vector mesons, the physical states are mixtures of octet and singlet

**Table 9.1** The $L = 0$ pseudoscalar and vector meson masses.

| Pseudoscalar mesons | | Vector mesons | |
|---|---|---|---|
| $\pi^0$ | 135 MeV | $\rho^0$ | 775 MeV |
| $\pi^\pm$ | 140 MeV | $\rho^\pm$ | 775 MeV |
| $K^\pm$ | 494 MeV | $K^{*\pm}$ | 892 MeV |
| $K^0, \bar{K}^0$ | 498 MeV | $K^{*0}/\bar{K}^{*0}$ | 896 MeV |
| $\eta$ | 548 MeV | $\omega$ | 783 MeV |
| $\eta'$ | 958 MeV | $\phi$ | 1020 MeV |

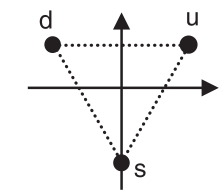
SU(3) flavour symmetry is **not** an excellent symmetry.

# Young tableaux for SU(3)

Extend the primitive object (of the fundamental 3 representation)

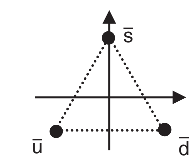
quarks

□ : [1], [2], [3]   $3$   (dimensionality 3, triplet)




antiquarks

[□]/[□] : [2]/[3], [1]/[2], [1]/[3]   $3^*$   (antisymmetric, dimensionality 3, antitriplet)




singlet

[□]/[□]/[□] : [1]/[2]/[3]   (singlet, totally antisymmetric state)

sextet

[□□] : [1 1], [1 2], [1 3], [2 2], [2 3], [3 3]   (symmetric, dimensionality 6, sextet)

decuplet

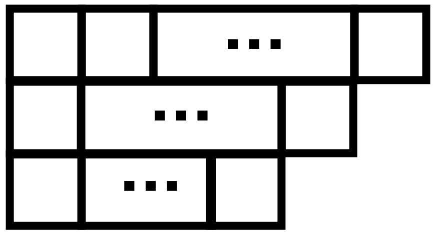
[□□□] : [1 1 1], [1 1 2], [1 2 2], [1 2 3] ...   (symmetric, dimensionality 10, decuplet)

octet

[□□]/[□] : [1 1]/[2], [1 2]/[2], [1 3]/[2], [1 1]/[3], [1 2]/[3], [1 3]/[3], [2 2]/[3], [2 3]/[3]   (mixed symmetry, dimensionality 8, octet)

# Young tableaux for SU(3)

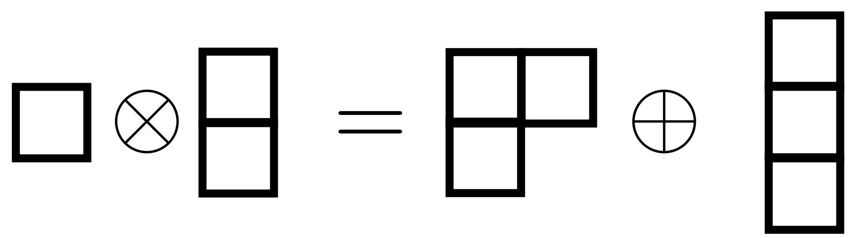
General rule for the dimensionality



$\lambda_1$ boxes
$\lambda_2$ boxes
$\lambda_3$ boxes

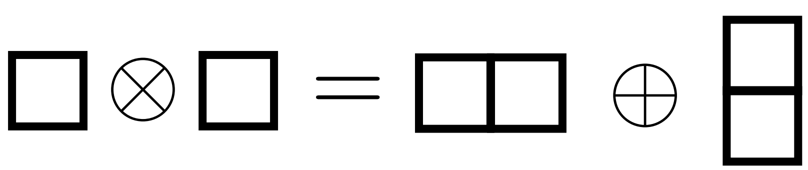
$$d = \frac{(p+1)(q+1)(p+q+2)}{2}; \qquad p = \lambda_1 - \lambda_2, \quad q = \lambda_2 - \lambda_3$$

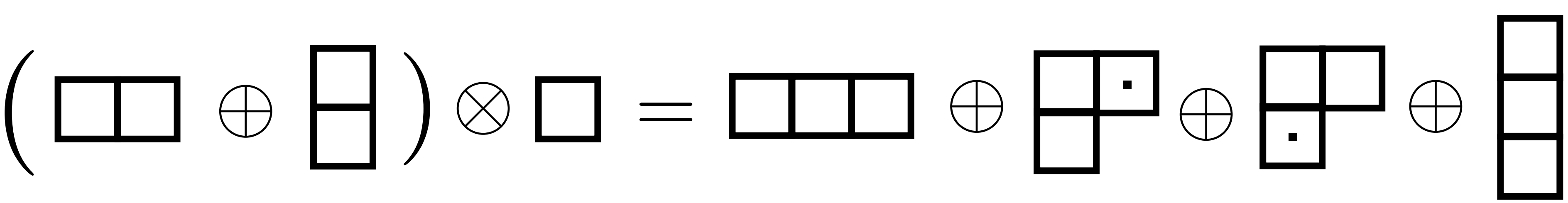
## Mesons

$3 \otimes 3^* =$  $= 8 \oplus 1$

## Baryons

$$3 \otimes 3 \otimes 3 = (3 \otimes 3) \otimes 3$$

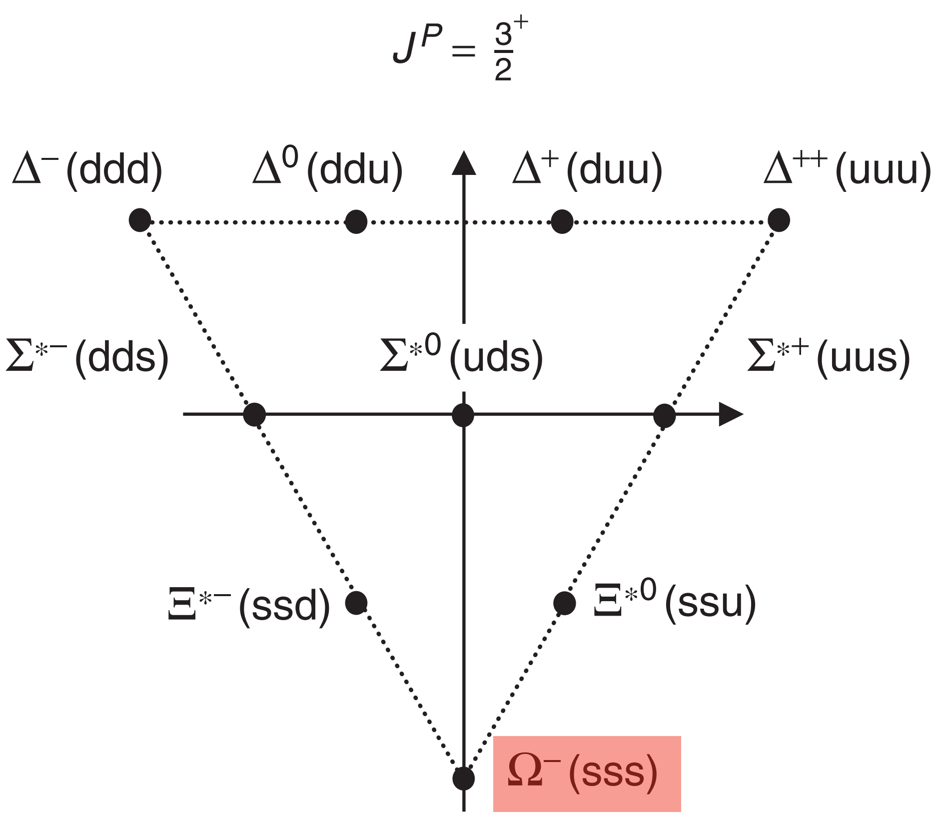
$3 \otimes 3 =$  $= 6 \oplus 3^*$

 $= 10 \oplus 8 \oplus 8 \oplus 1$

$$3 \otimes 3 \otimes 3 = (6 \oplus 3^*) \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

# Decuplet of L=0 baryons

We can take the symmetric decuplet and the symmetric s=+3/2 wave functions to make a decuplet of L = 0 baryons




m ~ 1230 MeV $Y = 1$

m ~ 1385 MeV $Y = 0$

m ~ 1583 MeV $Y = -1$

m ~ 1670 MeV $I_3 = 0, Y = -2$ $J = 3/2, Q = 1$

predicted in 1961 (Gell-Mann & Ne'eman)

discovered in 1964

# Octet of L=0 baryons

From the mixed symmetry octets one can make another octet of s=1/2 baryons

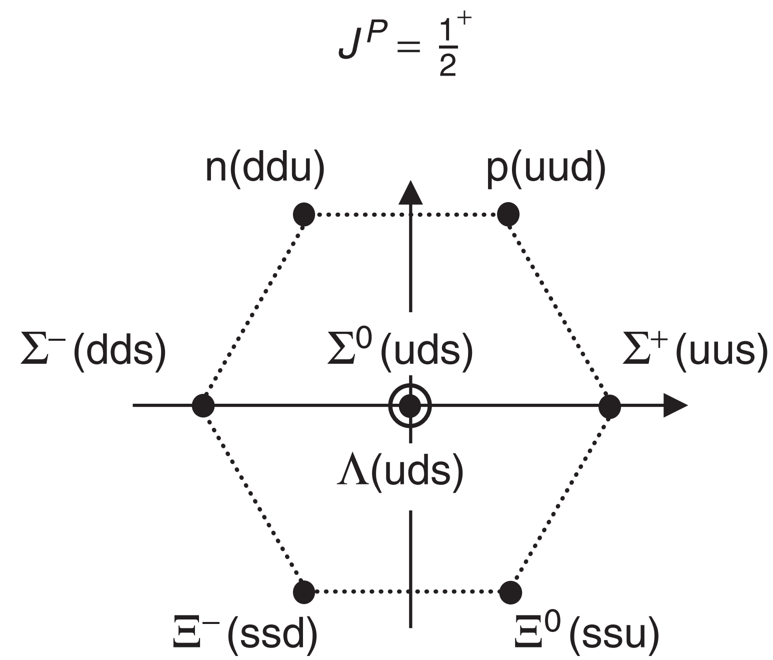



**Table 9.2** Measured masses and number of strange quarks for the $L = 0$ light baryons.

| s quarks | Octet | | Decuplet | |
|---|---|---|---|---|
| 0 | p, n | 940 MeV | $\Delta$ | 1230 MeV |
| 1 | $\Sigma$ | 1190 MeV | $\Sigma^*$ | 1385 MeV |
| 1 | $\Lambda$ | 1120 MeV | | |
| 2 | $\Xi$ | 1320 MeV | $\Xi^*$ | 1533 MeV |
| 3 | | | $\Omega$ | 1670 MeV |

# QCD: the need for colour

The quark model in SU(2) and SU(3) has nice features: understanding of hadronic states, classification, even a few predictions. *Fundamental degrees of freedom.*

## Problems of the quark model:

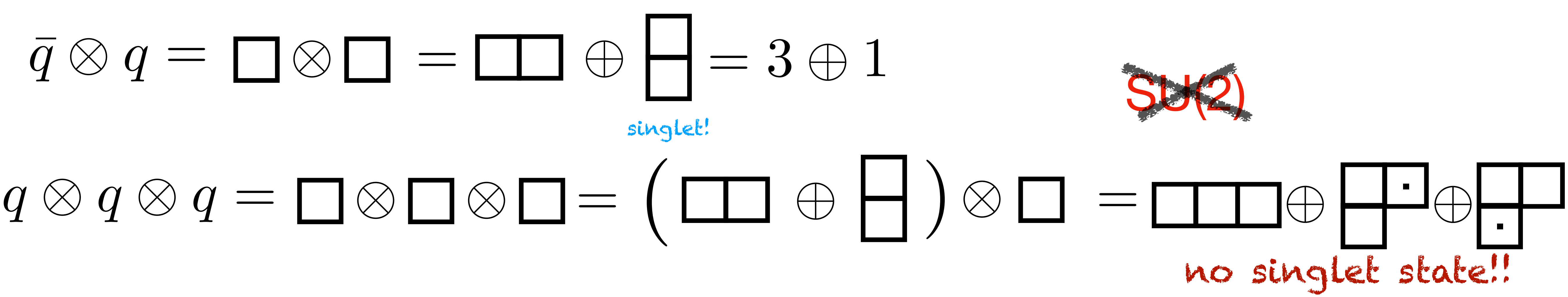
1. Hadrons have integer charges, but no reason to forbid qq or qqqq states.
2. Quarks were never seen in isolation
3. Symmetry of wave functions: without an additional antisymmetric component the baryon decuplet cannot be built (colour wave function).

**Solution:** additional quantum number (degree of freedom). New symmetry: rotations in color space.

> Only colour singlets (colourless states) are observed as hadrons.

Quark confinement!

How about adding an SU(2) quantum number to the quark field? Can we make singlet states?



# QCD: the need for colour



How about adding an **SU(3)** quantum number to the quark field? Can we make singlet states?

### Mesons (colour space)

$$q \otimes \bar{q} = 3 \otimes 3^* = \square \otimes \begin{array}{c}\square\\\square\end{array} = \begin{array}{ll}\square\square\\\square\end{array} \oplus \begin{array}{c}\square\\\square\\\square\end{array} = 8 \oplus \boxed{1}$$

### Baryons ($qqq$, colour space)

singlet!!

$$3 \otimes 3 \otimes 3 = (3 \otimes 3) \otimes 3$$

$$3 \otimes 3 = \square \otimes \square = \square\square \oplus \begin{array}{c}\square\\\square\end{array} = 6 \oplus 3^*$$

$$\left(\square\square \oplus \begin{array}{c}\square\\\square\end{array}\right) \otimes \square = \square\square\square \oplus \begin{array}{ll}\square\boxdot\\\square\end{array} \oplus \begin{array}{ll}\square\square\\\boxdot\end{array} \oplus \begin{array}{c}\square\\\square\\\square\end{array} = 10 \oplus 8 \oplus 8 \oplus \boxed{1}$$

### qq states are not allowed!

$$3 \otimes 3 = \square \otimes \square = \square\square \oplus \begin{array}{c}\square\\\square\end{array} = 6 \oplus 3^*$$

**q, qq, & qqqq states are not allowed (but qqq is allowed)!**

confinement of quarks

# QCD: the need for colour

Quarks have an additional quantum number: colour, which transforms under SU(3).

Hadrons are always colour singlet states!

Only colourless (singlet, or white) states can be observed! (Postulate)

This means that isolated quarks cannot be observed (nor qq or qqqq states for example)

Still, no dynamics: gauge principle is needed for that.

Confinement is still today, with QCD, not well understood.