ON THE HEAT-RADIATION OF LONG WAVE-LENGTH EMITTED BY BLACK BODIES AT DIFFERENT TEMPERATURES.[1]

By H. RUBENS and F. KURLBAUM.

As is well known, W. Wien[2] has derived, from thermodynamic considerations, the following formula, which gives the intensity, E, of the radiation of a black body for all wave-lengths, λ, and all temperatures, T,

$$E = C \frac{1}{\lambda^5} e^{-\frac{c}{\lambda T}} . \quad (1)$$

More recently Mr. Planck[3] has established Wien's law upon an electromagnetic basis, so that the subject has become one of increased interest.

Up to the present, two experimental investigations of Wien's formula have been undertaken, one by Lummer and Pringsheim,[4] the other by Paschen[5] working alone, and at a later date working with Wanner. In the region of shorter wave-lengths and lower temperatures, the agreement among the results of these observers is satisfactory, but as wave-lengths become longer and temperatures higher, the discrepancies become more considerable. For instance, while Paschen always obtains exact agreement between his observations and Wien's formula, Lummer and Pringsheim find that for sufficiently high values of the product λT the deviations from this formula are very considerable. The contrast between theory and experiment is especially marked when one considers the so-called isochromatic curves, which express the intensity of radiation as a function of the

[1] From the *Sitzungsberichte der Akad. Wiss. Berlin*, Oct. 25, 1900.

[2] W. WIEN, *Wied. Ann.*, **58**, 662, 1896.

[3] M. PLANCK, *Sitzungsberichte Ber. Akad.*, 1899, p. 440.

[4] O. LUMMER and E. PRINGSHEIM, *Verhandlung der Deutschen Phys. Ges.*, I. Jahrg. S. 23 and 215, 1889; II. Jahrg. S. 163, 1900.

[5] F. PASCHEN, *Wied. Ann.*, **58**, 455, 1896; **60**, 662, 1897; *Berichte Berl. Akad.*, 1899, 405 and 959; *ibid.*, F. PASCHEN and H. WANNER, p. 5.

temperature for any given wave-length. The equation of such an isochromatic curve is, according to Wien,

$$E = \text{const.}\; e^{-\frac{c}{\lambda T}} \;.$$

In order to represent their observations by this equation, Lummer and Pringsheim were compelled to assign a variable value to the quantity c, namely for

$\lambda = 1.2\mu$	2μ	3μ	4μ	5μ
$c = 13{,}900$	14,500	15,000	15,400	16,400

For still greater wave-lengths, it was found impossible to give an even approximate description of the isochromatic curve by means of a simple exponential function. For instance, the isochromatic curve for $\lambda = 12.3\mu$ calls for values of c which range from 14,200 to 24,000 as the temperature rises; while for $\lambda = 17.9\mu$ the values of c vary from 17,200 to 27,600.

Since, now, the quantity c enters Wien's expression as an absolute constant, it is evident from the experiments of Lummer and Pringsheim that this formula is not capable of describing the facts for longer wave-lengths and higher temperatures.

Thiesen[1] has recently proposed an empirical formula, which is based upon the observations of Lummer and Pringsheim for shorter wave-lengths ($\lambda < 7\mu$), and which appears to fit the facts much better than the law proposed by Wien.

Thiesen's expression is

$$E = C \cdot \frac{1}{\lambda^5} \cdot \sqrt{\lambda T} \cdot e^{-\frac{c}{\lambda T}} \;. \tag{2}$$

The one point of difference between this and Wien's equation is the presence of the factor $\sqrt{\lambda T}$.

Some months ago Lord Rayleigh[2] also discussed Wien's law of radiation, and pointed out the fact that it is inherently improbable, because for each wave-length it gives only finite values of intensity for infinite values of temperature. Rayleigh then proposed as a substitute

$$E = C \cdot \frac{1}{\lambda^5} \cdot \lambda T \cdot e^{-\frac{c}{\lambda T}} \;. \tag{3}$$

[1] M. THIESEN, *Verhandlungen der Deutschen Phys. Ges.*, **2**, 37, 1900.

[2] RAYLEIGH, *Phil. Mag.*, **49**, 539, 1900.

Still a fourth general formula which includes the previously mentioned ones as special cases has been recently published by Lummer and Jahnke.[1] It runs as follows:

$$E = C \cdot \lambda^{-\mu}\ T^{5-\mu} \cdot e^{-\frac{c}{(\lambda T)^{\nu}}}\ . \qquad (4)$$

Lummer and Pringsheim find that all of their observations which lie between $\lambda = 1\mu$ and $\lambda = 18\mu$ are in excellent agreement with this formula when $\mu = 4$ and $\nu = 1.3$. The difference between this expression and Lord Rayleigh's lies in the factor ν, which, in Rayleigh's equation, has the value unity. We have, therefore

$$E = C \cdot \frac{1}{\lambda^5} \cdot \lambda T \cdot e^{-\frac{c}{(\lambda T)^{1.3}}}\ . \qquad (4a)$$

Finally, Planck,[2] since the completion of our experiments, has brought out a fifth formula, namely:

$$E = C \cdot \frac{\lambda^{-5}}{e^{\frac{c}{\lambda T}} - 1}\ . \qquad (5)$$

For short wave-lengths and low temperatures this expression approaches Wien's; for long waves and high temperatures it is more nearly equivalent to Lord Rayleigh's; while it includes both as limiting cases.

Each of these equations, like that of Wien, implies Stefan's law of radiation, and also the following two relations,[3] which have been established by various observers, $\lambda_m T =$ constant and $\frac{E_{\text{max.}}}{T^5} =$ constant.[4]

[1] O. LUMMER and E. JAHNKE, *Drude's Ann.*, **3**, 283, 1900.

[2] M. PLANCK, *Berichte der Deutschen Phys. Ges.*, **2**, Oct. 19, 1900.

[3] M. THIESEN, *ibid.*

[4] In each of the six equations given above the constant c has a different value, namely:

$$\left.\begin{array}{ll}\text{In Equation (1),} & c = 5\ (\lambda_m T) \\ \text{In Equation (2),} & c = 4.5\ (\lambda_m T) \\ \text{In Equation (3),} & c = 4\ (\lambda_m T) \\ \text{In Equation (4),} & c = \frac{\mu}{\nu}\ (\lambda_m T)^{\nu} \\ \text{In Equation (4a),} & c = \frac{1}{1.3}\ (\lambda_\mu\ T)^{1.3} \\ \text{In Equation (5),} & c = 4.965\ (\lambda_m T)\end{array}\right\}\ \lambda_m T = 2890.$$

For small values of the product λT, all of these formulæ give nearly the same series of values for E; but for high temperatures and long wave-lengths the differences which characterize these various equations show themselves in a marked manner. In this case, for instance, the exponential quantities $e^{-\frac{c}{\lambda T}}$ and $e^{-\frac{c}{(\lambda T)^\nu}}$ approach unity, and we have for the isochromatic curve, according to Wien, E=const.; according to Thiesen, E=const. $\sqrt{T}$; and according to Rayleigh, Lummer-Jahnke, and Planck, E=const.T. Now, in view of the fact that exact measurements are limited to temperatures less than 1500° C., it is evident that this case cannot be realized experimentally, that is, we cannot pass to wave-lengths so large, and to temperatures so high that the effect of the exponential quantity will completely disappear. Not only so, but there is a limit to the length of wave which can be measured with sufficient accuracy. However, it is always possible to go much farther in this direction by using the method of residual[1] rays than by ordinary processes of dispersion. We are, therefore, in a position to determine the fitness of these various formulæ in the region of larger wave-lengths.

At the suggestion of one of us, some measurements of this kind were carried out not long ago by Mr. Beckmann.[2]

He allowed the radiation from a black body to undergo reflection at four fluorite surfaces and then measured the intensity of the residual rays for various temperatures of the radiating black body.

As was shown not long since, there is a region in the infra-red — rather sharply limited — where fluorspar exhibits metallic reflection and in which two maxima occur, one at $\lambda = 24\ \mu$, the other at $\lambda = 31.6\ \mu$.

Experiment proves that after four reflections at fluorite surfaces there remain only such radiations of the black body as

[1] On the subject of residual rays, their production and their properties, see H. Rubens and E. F. Nichols, *Wied. Ann.*, **60**, 418, 1897; H. Rubens and E. Aschkinass, *Wied. Ann.*, **65**, 241, 1898; and H. Rubens, *Wied. Ann.*, **69**, 576, 1899.

[2] H. Beckmann, *Inaug. Dissert.*, Tübingen, 1898.

belong to the region of metallic reflection. This bundle of rays shows maxima of intensity at $\lambda = 24.0\,\mu$ and at $\lambda = 31.6\,\mu$. For the purpose of comparing observations with the above mentioned formulæ, we may assume that all the residual rays consist of two perfectly homogeneous radiations, one whose wave-length is $24.0\,\mu$, the other $31.6\,\mu$. Besides this, we must consider that the reflecting power of each fluorite surface at $\lambda = 21.6\mu$ is nearly 1.2 times as great as at $\lambda = 24.0\,\mu$, so that the relative intensity of the second band compared with the first is increased in the ratio $1.2^4 = 2.0$.

Beckmann, independently of the experiments of Lummer and Pringsheim, inferred from his own observations that Wien's formula could not correctly represent the facts by giving c the value 14500, which it has for short wave-lengths.

In order to obtain agreement between observed and computed values it was necessary to place[1] $c = 26000$. It was impossible for Beckmann to compare his results with the predictions of any others of the formulæ discussed in our introduction for the reason that these formulæ had not then been published.

And, indeed, Beckmann's observations were not very well adapted to test this law because the interval between his extreme temperatures is too small. The measurements begin at the temperature of solid carbon dioxide and end at about 600° C. While, as I have pointed out above, the characteristic features of these equations become marked only when we reach temperatures outside of this region and especially temperatures above this region.

We have therefore undertaken to measure the intensity of the residual rays from a black body throughout the largest possible range of temperatures. This research was made to include not only the residual rays of fluorspar but also those of rock salt which have a mean wave-length of $51.2\,\mu$. In this way we reached values of the product λT which are three times as

[1] H. Rubens, *ibid.*, p. 585. The fact that Beckmann was able to represent his observations by one of Wien's isochromatic curves is explained by the limited range of his temperatures.

great as those hitherto obtained by means of spectroscopic separation. The diagram given in Fig. 1 shows the disposition of our apparatus.

D_1 is a double-walled diaphragm through which flows water at the temperature of the room, 20° C. The diaphragm is circular in form and the aperture one centimeter in diameter. This is mounted firmly upon the table and forms what is practically the source of radiation.

In front of this diaphragm is placed a black body R in such

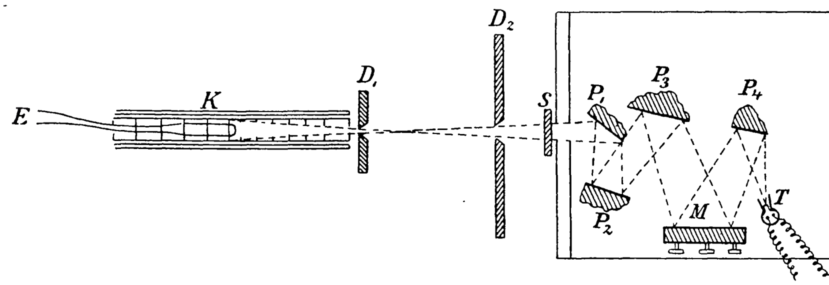

FIG. 1.

a way that its open end just fits into the aperture in the diaphragm and its axis stands at right angles to the plane of the diaphragm, adjustments which are secured by optical and mechanical devices. The rays which pass through D_1 traverse also a second diaphragm D_2, which limits the cone of rays in such a way that only those from the innermost part of the black body can get through. Farther on in the path of the rays is placed a double-walled screen, through which also flows water from the same supply as that in D. Still farther on in the path of the rays are the reflecting surfaces, P, of fluorspar or rock salt, as the case may be, a condensing mirror, M, silvered on the front, and the thermopile, T.[1] These latter parts of the apparatus are protected against outside radiation and air currents by being placed in a case. The thermopile was connected to a galvanometer of the shielded[2] form ("*Panzer Galvanometer*") whose sensibility was constantly under control by means of a simple

[1] H. RUBENS, *Zeitschrift für Instrumentenkunde*, **18**, 65, 1898.

[2] H. DU BOIS and RUBENS, *Drude's Ann.*, **2**, 84, 1900.

device. Changes in sensibility were always taken into account in the computation of our results.

The impurities of the residual rays consist of heat rays which, in composition, are practically the same as the total radition of the black body. Accordingly the intensity of the impurities must approximately obey Stefan's law and increase with the fourth power of the temperature, while the intensity of the residual rays varies directly as the first power of the temperature. It follows, therefore, that the relative impurity increases as the third power of the absolute temperature of the black body.

While four reflecting surfaces were found sufficient to isolate the residual rays of fluorspar with a fair degree of purity, it was discovered that this number of surfaces was not competent to separate the very weak residual rays of rock salt. By the use of five surfaces we obtained residual rays of satisfactory purity up to temperatures of 600° C., at which the impurity due to ordinary heat radiation amounted to 10 per cent. By the introduction of a sixth rock salt surface the impurity was reduced to about $\frac{1}{20}$. But at temperatures higher than 1000° C. it was again marked and at the highest attainable temperature 1474° C. amounted to almost 8 per cent. of the quantity being measured. We did not therefore attempt to further increase the number of reflecting surfaces, but preferred rather to determine exactly what correction was necessary to compensate for the impurity of the radiation. This was done by means of a rock salt plate which completely absorbed the residual rays while it transmitted 90 per cent. of the impurity.

In the experiments with fluorspar we employed four different black bodies which had already been used in other investigations at the Reichsanstalt.[1] The first of these (I) was a hollow radiating body so arranged as to be cooled by a stream of liquid air flowing over it. The second (II) was so arranged that it could be filled with solid carbon dioxide and ether. The

[1] O. LUMMER and F. KURLBAUM, *Verhandlungen der Berliner Phys. Ges.*, 17, 106, 1898; and *Thätigkeitsbericht der Phys. Tech. Reichsanstalt*, p. 38. 1899.

third (III) was heated by steam, and the fourth (IV) by an electric current. This last was the only one used in the fluorspar experiments between 300° C. and 1500° C. In order to get the exceedingly feeble residual rays of rock salt, especially at low temperature and with sufficient accuracy, we removed the diaphragm D_1 (Fig. 1), and placed successively the first three of the black bodies immediately in front of the diaphragm D_2.

This was admissible since these three black bodies each possessed an aperture greater than that of D_2. The electrically heated black body (IV) had an aperture of only 12 mm, so that we were compelled in this case to use also the diaphragm D_1 as indicated in Fig. 1.

We have therefore constructed for this investigation two more electrically heated black bodies which, like bodies I, II, and III, have sufficiently large linear apertures (30 mm) and emit sufficiently large cones of rays to be used immediately in front of diaphragm D_2.

One of these (V) was made of "Marquardt's substance" wrapped with a platinum band and could be used in the region of temperatures lying between 300° C. and 1500° C. The other (VI) was made of iron, blackened with iron oxide, and was heated by means of an electric current passing through a spiral of nickel. The highest temperature to which this body could be heated was 600° C. Accordingly it has been used only between temperatures 300° and 600° C.

As noted above, the black body IV was employed, in connection with the diaphragm D_1, for temperatures higher than 500° C. The deflection thus obtained was 7.5 times smaller than that obtained from bodies V and VI placed in front of diaphragm D_2. Deflections obtained with the body IV were therefore multiplied by this factor in order to make them comparable with other observations.

This numerical factor was determined by making the deflections due to bodies IV and V equal at a given temperature, approximately 1000°. On account of the smallness of the deflections produced by IV it is evident that the observations made on this body are much less accurate than the others. Nevertheless they are valuable as checks.

In Fig. 3 the points obtained by observation upon each of these different bodies are indicated by a different mark.

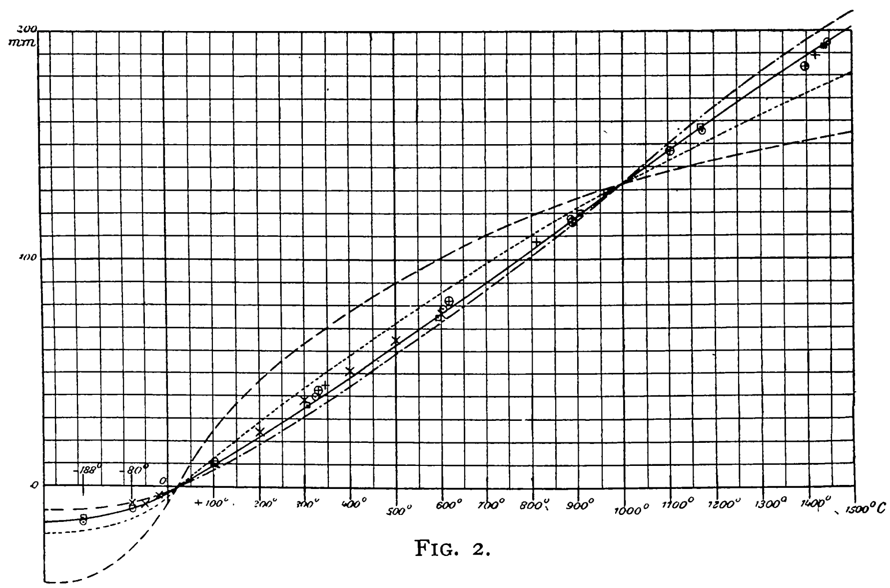

FIG. 2.

- - - - - $E = f(t)$ calculated according to Wien.
.... $E = f(t)$ calculated according to Thiesen.
-.-.-.- $E = f(t)$ calculated according to Lord Rayleigh.
———— $E = f(t)$ calculated according to Planck.

× × × Observations of Beckmann.
⊕ ⊕ ⊕ Observations of Rubens and Kurlbaum with sylvine plate.
+ + + / ⊙ ⊙ ⊙ / ⊡ ⊡ ⊡ Series of observations of Rubens and Kurlbaum without sylvine plate with different adjustments of the fluorite surfaces.

In the case of the electrically heated bodies the temperatures were determined as usual by a Le Chatelier thermopile (E, Fig. 1) based upon the latest results of Holborn and Day.[1]

In Fig. 2 are shown the results of our observations on the residual rays of fluorspar, and in Fig. 3 those for rock salt, that is, the observed deflections are plotted as a function of the temperatures of the radiant bodies. And, by the use of different signs,

[1] L. HOLBORN and A. DAY, *Wied. Ann.*, **68**, 817, 1899.

four entirely independent sets of observations, made on different days and after independent adjustment of the fluorspar surfaces, are represented in Fig. 2.

In one of these sets where the individual observations are indicated by a point surrounded by a small circle (thus ⊙) a plate

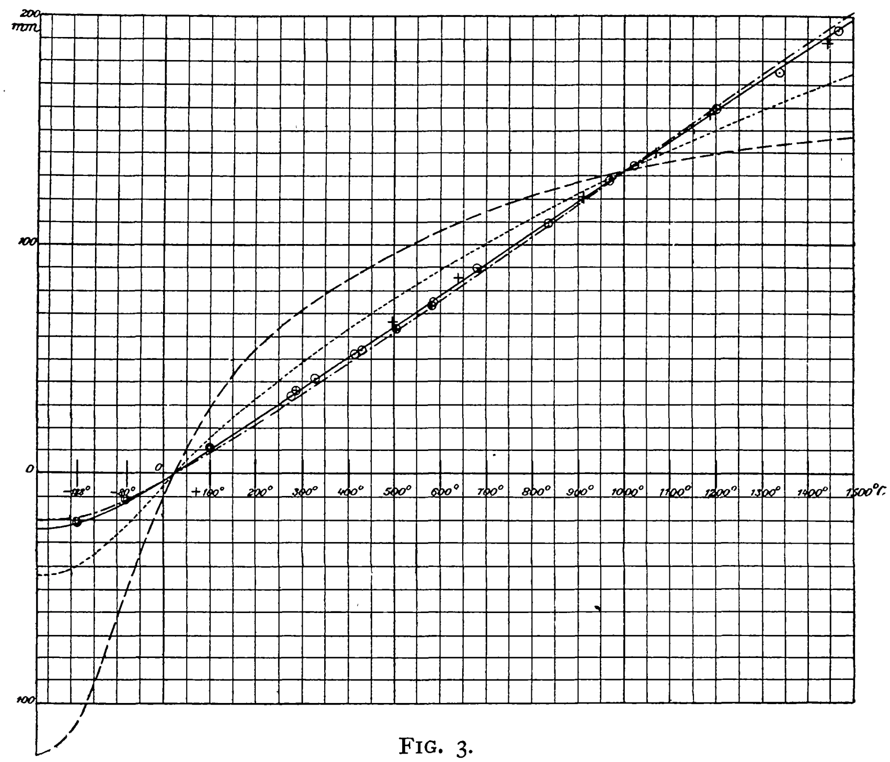

FIG. 3.

_ _ _ _ _ $E = f(t)$ calculated according to Wien.
.... $E = f(t)$ calculated according to Thiesen.
_ _ . _ . _ $E = f(t)$ calculated according to Lord Rayleigh.
——— $E = f(t)$ calculated according to Planck.

⊕ ⊕ ⊕ Observed with Black Bodies I, II, and III.
⊙ ⊙ ⊙ Observed with Black Body IV.
\+ \+ \+ Observed with Black Body V.
◎ ◎ ◎ Observed with Black Body VI.

of sylvine 2 mm thick was placed in the path of the rays immediately in front of the thermopile. This served to completely absorb the long waves among the residual rays while it transmitted[1] nearly half of that radiation which has its maximum at $\lambda = 24.0\mu$.

In this manner the influence of the second absorption band was completely eliminated. Yet the results of this series show that the intensity of radiation varies with temperature in practically the same manner as when the plate of sylvine is not used.

This series coincides however more exactly with the others when each ordinate is multiplied by the constant factor 2.80, as has been done in Fig. 2.

Finally a smooth curve — the solid line in Fig. 2 — was drawn connecting all the points determined by observation. In the region of low temperatures ($-188°$ to $0°$) this curve exhibits marked curvature, being concave on the upper side. Farther on this curvature disappears almost completely and the curve becomes rectilinear. In other words, the residual rays between 0° and 1500° increase in direct proportion to the difference of temperature between the body which emits and the body which absorbs the radiation. The same is also true for rock salt, as is evident from Fig. 3, where the curve also begins with a slight concavity on the upper side and soon becomes rectilinear. For the sake of more easily comparing the curves of Figs. 2 and 3, we have given them each the same ordinate[2] at the temperature of 1000° C. And, as is evident, they differ very little for other temperatures.

In order to make possible a comparison of our results with those of Beckmann we have reduced his results to our scale and have plotted them in Fig. 2, where they are indicated by an asterisk (thus *). The agreement, as will be seen, is very satisfactory: and this coincidence is all the more important, since Beckmann's observations were confined to one black body while ours in the interval in question ($-80°$ to $600°$) were made upon three different bodies. It may therefore be considered as proved that the various bodies employed by us behave in the

[1] H. RUBENS and A. TROWBRIDGE, *Wied. Ann.*, **60**, 724, 1897; also H. RUBENS and E. ASCHKINASS, *Wied. Ann.*, **65**, 253, 1898.

[2] The numerical value of the ordinates in the figures are so chosen that they are very nearly the actual deflection in millimeters produced by the residual rays of rock salt.

same way so far as an approximation to Kirchhoff's ideal black body is concerned. From Fig. 3, the same appears to be true also for rock salt. For here the black bodies IV, V, and VI give, within the limits of error, the same deflections throughout the range of temperatures in which they could be compared namely, from 275° to 600° and from 500° to 1500°.

Besides the results of direct observation, Figs. 2 and 3 contain also three other curves showing the dependence of residual rays upon temperature, according to the formulæ proposed by Wien, Thiesen, and Lord Rayleigh. A fifth curve exhibiting this same function according to the formula of Lummer and Jahnke, using the constants $\mu = 4$ and $\nu = 1.3$, can be shown only for the lower temperatures, since it coincides almost perfectly with our observed curve. This is the formula which Messrs. Lummer and Pringsheim employed to represent their work. For the same reason it is impracticable to plot Planck's formula (5) in Figs. 2 and 3, since his expression agrees perfectly with our observations, not only from 0° to 1500,° but also from −188° to 0°, at least to within errors of observation. The small deviations between our experimental results and the predictions of formulæ (4^a) and (5) may easily be seen from the following tables. The scales of all the curves are so chosen as to make the ordinates at 1000° exactly the same. In the computation of the curves for Fig. 2 we have always corrected for the presence of the band at $\lambda = 31.6\mu$, although the form of the curve would scarcely be affected if we were to neglect it entirely, and assume that the radiation is confined to a single band at $\lambda = 24\mu$. For temperatures above 0°, these deviations could scarcely be seen on the scale chosen for Fig. 2, since they are all less than 1 mm. They amount to an appreciable quantity only for very low temperatures, but for the sake of avoiding complication they are not shown in the diagram.

A glance at these curves is sufficient to prove that none of the formulæ of Wien, Thiesen and Rayleigh is capable of describing the results of observation within the limits of experimental error. Rayleigh's formula fits our results most closely, while that of Wein is the least adapted.[1] On the other hand, the deviations of our figures from the formula of Lummer and Jahnke (4^a) are very slight. These deviations become as great as the errors of observation only in the case of very low temperatures, where the deflections are 20 per cent. smaller than the computed values. For temperatures of the black body between 0° and 1500° C. the coincidence is perfect. We have already called attention to the fact that Planck's formula (5) describes our experiments for all temperatures.

In the two following tables are collected the interpolated values of the observed deflections for various temperatures, together with the values predicted by the formulæ (1), (2), (3), (4^a), and (5) both for the case of fluorspar and for the case of rock salt.

The most marked difference between formula and experiment is in the case of Wien's values for the residual rays of rock salt. At the temperature of liquid air the deflection observed is only about one-fifth of that computed, while, on the other hand, the deflection observed (194 mm) at 1474° is the

TABLE I.

Residual rays of fluorspar, $\lambda = 24.0\mu$ and 31.6μ.

Temperature Centigrade t	Absolute Temperature T	E Obs.	E according to Wien	E according to Thiesen	E according to Rayleigh	E according to Lummer and Jahnke	E according to Planck
− 273	0	....	− 42.4	− 20.7	− 10.7	− 17.8	− 15.4
− 188	85	− 15.5	− 41.0	− 20.2	− 10.5	− 17.5	− 15.0
− 80	193	− 9.4	− 26.8	− 14.0	− 7.4	− 11.5	− 9.3
+ 20	293	0	0	0	0	0	0
+ 250	523	+ 30.3	+ 50.6	+ 35.7	+ 25.3	+ 30.0	+ 28.8
+ 500	773	+ 64.3	+ 88.9	+ 71.8	+ 58.3	+ 64.5	+ 62.5
+ 750	1023	+ 98.3	+114	+104	+ 94.4	+ 98	+ 96.7
+1000	1273	+132	+132	+132	+132	+132	+132
+1250	1523	+167	+145	+157.5	+174.5	+167	+167.5
+1500	1773	+201.5	+155	+181	+209	+201	+202
+ ∞	∞	....	+226	+ ∞	+ ∞	+ ∞	+ ∞

[1] The single point of intersection chosen for these curves, namely, $t = 1000°$ C. is selected for the purpose of making the divergence between theory and experiment as small as possible. If these curves had been made to coincide at $t = 1500°$ C., the discrepancy would have been much more marked.

TABLE II.

Residual rays of rock salt, $\lambda = 51.2\mu$.

Temperature Centigrade t	Absolute Temperature T	E Obs.	E according to Wien	E according to Thiesen	E according to Rayleigh	E according to Lummer and Jahnke	E according to Planck
− 273	0	....	−121.5	− 44	− 20	− 27	− 23.8
− 188	85	− 20.6	−107.5	− 40	− 19	− 24.5	− 21.9
− 80	193	− 11.8	− 48.0	− 21.5	− 11.5	− 13.5	− 12.0
+ 20	293	0	0	0	0	0	0
+ 250	523	+ 31.0	+ 63.5	+ 40.5	+ 28.5	+ 31	+ 30.4
+ 500	773	+ 64.5	+ 96	+ 77	+ 62.5	+ 65.5	+ 63.8
+ 750	1023	+ 98.1	+118	+106	+ 97	+ 99	+ 97.2
+1000	1273	+132	+132	+132	+132	+132	+132
+1250	1523	+164.5	+141	+154	+167	+165.5	+166
+1500	1773	+196.8	+147.5	+175	+202	+198	+200
+ ∞	∞	....	+194	+ ∞	+ ∞	+ ∞	+ ∞

limiting value set by Wein's formula for an infinitely high temperature, assuming the scale which we have here employed.

In any event, it is evident from the preceding results that only those formulæ which make the radiation E vary directly as the temperature T, are competent to describe the behavior of a black body for large wave-lengths and high temperatures. Such formulæ are those of Lord Rayleigh, Lummer-Jahnke ($\mu = 4$) and Planck.

Of these three formulæ, however, only the last two are to be considered, since Lummer and Pringsheim have shown that Rayleigh's expression does not represent the facts in the case of short wave-lengths. In comparison with our observations, also, it shows considerable systematic deviation. We find, therefore, that so far as the representation of Lummer and Pringsheim's results, as well as our own, is concerned, the formulæ (4^a) and (5) are excellently adapted, but that Planck's expression, in so far as it does the same thing, deserves the preference on account of its simplicity.

[Shortly after the publication of this article in the *Proceedings of the Berlin Academy*, it appeared in a somewhat amplified form in *Drude's Annalen*, **4**, 649–666 (1901). The principal change from the present article consists in addition of an isochromatic curve for wave-length $\lambda = 8.85\mu$, determined by the measurement of residual rays reflected from quartz. The isochromatic thus obtained confirms the conclusions which the authors had already drawn from their observations on rock salt and fluorspar.]—ED.