

Faraday Effect

Goal: Magneto-optics: optical activity in a homogeneous medium in the presence of a magnetic field. Measure the material constant that quantifies this effect (Verdet constant)

Equipment: Diode laser, coil, controllable current supply, wave generator, photo diode, amplifier, storage scope...

1 Introduction

In 1854 M. Faraday found that isotropic media become optically active when a magnetic field is applied in the propagation direction of the light. This was one of the earliest indications that light and electro-magnetism are related. The angle ϕ by which the plane of polarization of a linearly polarized beam is rotated is given by

$$\phi = V \cdot \int B dl \quad (\text{radians}) \quad , \quad (1)$$

where V is the so-called Verdet constant, and B is the magnetic field in the direction of the light, and the integral extends over the full length of the medium (in our case a cuvette with water).

In order to understand the effect, it is useful to think of linearly polarized light as the superposition of left- and right-handed circularly polarized light of equal amplitude. If it happens that the index of refraction for left and right light are not the same (this is called circular bi-refringence), then the phase of one component will advance relative to the other by some angle. When recombining the two components, linear polarization results that is rotated by half that angle.

Quantum mechanics is needed to understand the reason for the different indices of refraction. Left and right light is really a beam of polarized photons with spin 1 and magnetic substate +1 or -1, respectively. In a magnetic field, the energy levels that can be excited by such photons correspond to slightly different frequencies (discovered by Zeeman). The frequency shift is the Larmor frequency $\omega_L = (e/2m)B$, where e and m are charge and mass of the electron. From this it is quite straightforward to derive the following expression for the Verdet constant

$$V = -\frac{e}{2mc} \lambda \frac{dn}{d\lambda} \quad . \quad (2)$$

This equation, which was derived by H. Becquerel (1897) shows that the Verdet constant depends on the wavelength, the dispersion, and the charge to mass ratio of the electron. A theoretical treatment of Faraday rotation can be found in Ref. [MOL88], p.299, as well as in Ref. [MON63]. A measurement of the Verdet constant similar to the one carried out in this lab is treated in Ref. [PRE91], p.355; you should carefully study this description.

2 Magnetic Field

The magnetic field is produced in a long solenoid. The solenoid has six layers of windings of #16 wire. The number of turns are (331+318+351+332+355+336), adding to a total of $N = 2023$ turns. The last layer stops roughly 11 turns short of the full length of the coil. The electrical resistance of the coil has been measured to be 3.72 Ohms at room temperature (note that this increases with increasing temperature).

The length of the coil (47 cm) is quite a bit shorter than the length of cuvette (61 cm). We may thus assume that the field B has fallen off to a negligible value before the end of the cuvette. In this case, the integral in eq. 1 may as well be taken from $-\infty$ to $+\infty$. From E&M, we know that

$$\int_{-\infty}^{+\infty} B dl = \mu_0 N I \quad , \quad (3)$$

where $\mu_0 = 4\pi \cdot 10^{-7} \text{ mkg/C}^2$, N is the number of windings, and I is the current through the coil.

3 Measurements

3.1 Magnetic field

The only reason why a measurement of a magnetic field is required is to estimate the error that is made when using the field integral to infinity as in eq.3. In order to determine what fraction of the field is outside the sample tube, a measurement of the B field along the axis of the solenoid is required, covering the range from well within the coil to well beyond the end of the sample tube.

3.2 Laser and light detection

A red and green diode laser ($\lambda = 642.2, 531.9 \text{ nm}$, respectively) are available for this measurement. The laser output intensity exhibits slow variations in time. One has to keep this in mind for the following measurements. As the laser is warming up, these variations diminish. Measuring the light intensity as a function of time (with no other change) gives you a handle on this effect.

The laser beam should be aligned carefully, centering it on all optical components, the sample tube, and the photodiode. If the analyzing Nicol prism is not orthogonal to the beam, rotating it displaces the light beam on the photodiode. If the beam is not centered well on the photodiode, there may be an unwanted intensity change. In addition, the signal from the photodiode may not be linear in intensity. In order to test the alignment and linearity, measure the intensity as a function of the analyzer angle φ for a full revolution, and compare with the expected $\sin^2\varphi$ dependence.

The resistive heating of the magnet coil heats the sample and produce convection currents with varying index of refraction. Make sure the coil cooling is on.

3.3 Verdet constant

The method used in this experiment is to observe the intensity change of the transmitted light due to a current change of ΔI . In a second measurement one then determines the angle change $\Delta\varphi$ of the analyzing Nicol prism that causes the same

intensity change.

The measurement should be carried out for a range of magnetic fields in order to demonstrate that $\Delta\phi$ is proportional to ΔI . Eqs. 1 and 3 are then used to derive the Verdet constant. Systematic and random errors of the measurement should be discussed.

The measurement should be carried out at two different wavelengths, to be able to verify Becquerel's equation.

Historically, the Verdet constant is quoted in units of arc-minutes/Gauss/cm. To convert to S.I. units, convince yourself that $1 \text{ min/Gauss/cm} = 290.9 \text{ radians/T/m}$.

The Verdet constant for water may depend on impurities in solution. It is thus necessary to use pure, distilled water for this measurement.

3.4 Possible extensions

It may be interesting to measure the Verdet constant for a salt solution and determine how it depends on the concentration. A bottle of pure NaCl is available for this purpose.

4 Analysis

4.1 Comparison to literature values of the Verdet constant

Some tabulations of Verdet constants are given in ref. [MON63], p.314, and ref. [MOL88], p.300. The most comprehensive collection of data can be found in ref. [LANBI]. The relevant part of the summary figure from that reference is reproduced in fig.1.

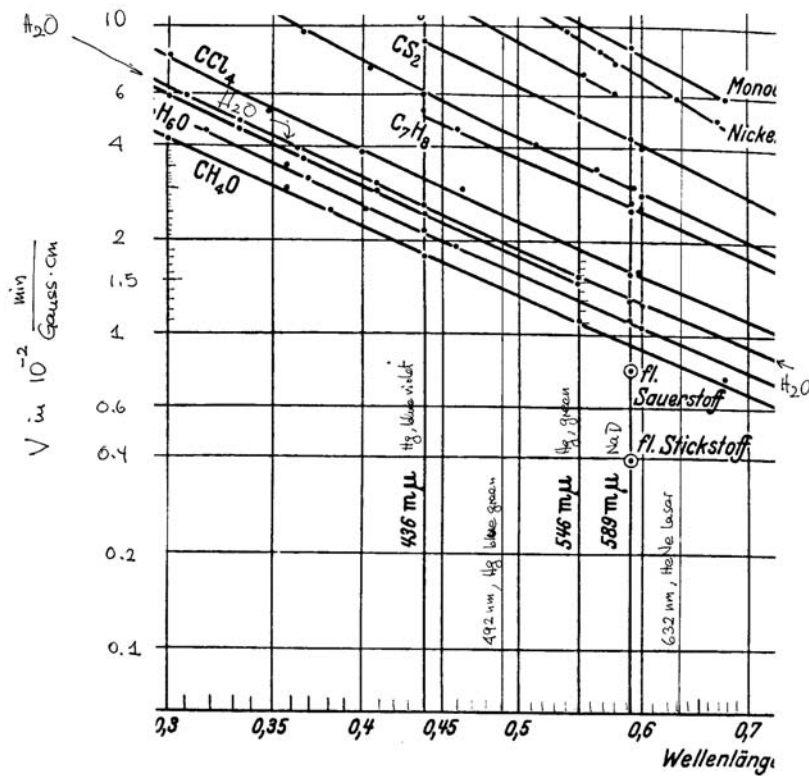


Fig.1: Verdet constant as a function of wavelength for a number of liquids (from ref. [LANBI])

4.2 Becquerel's equation (Eq. 2)

Eq. 2 states that V is proportional to $\lambda(dn/d\lambda)$, with the constant of proportionality made up from fundamental constants.

The index of refraction $n(\lambda)$ for water can be found in the literature. The data can be fit with Cauchy's expansion

$$n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} \quad , \quad (4)$$

which produces values for A, B, and C. Taking the derivative and multiplying with λ , leads to fig.2. Try to verify this, starting from literature values.

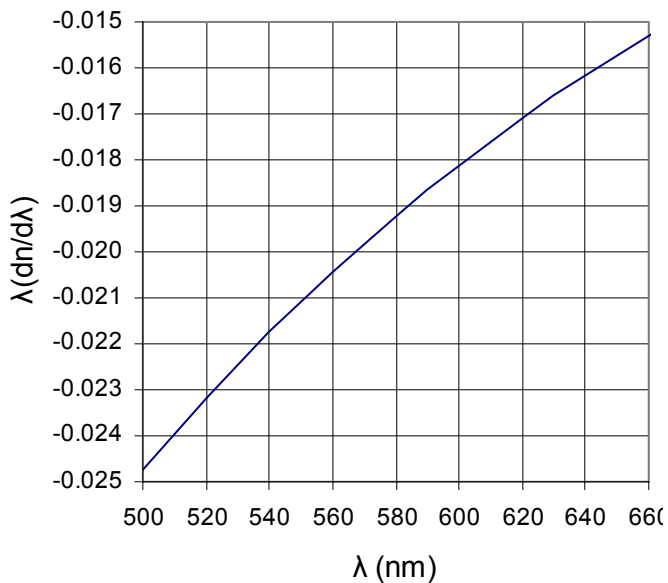


Fig. 2: Wavelength times dispersion, $\lambda(dn/d\lambda)$, for water at 20°C.

5 References

- [MON63] G.S. Monk, Light: Principles and Experiments, Dover, New York 1963 (QC357.M7)
- [MOL88] K.D. Moller, Optics, Univ. Sci. Books, Mill Valley, CA, 1988 (QC355.2)
- [PRE91] D.W. Preston and E.R. Dietz, The Art of Experimental Physics, John Wiley, New York 1991.
- [LANBI] Landolt- Biörnstein, Physics Data Tables, 6th edition, QC61.125, Vol. I/1, p.406

