5.1.10-05/07 Zeeman effect/normal and anomalous version

What you can learn about …

- → Bohr's atomic model
- → Quantisation of energy levels
- → Electron spin
- → Bohr's magneton
- → Interference of electromagnetic waves
- → Fabry-Perot interferometer

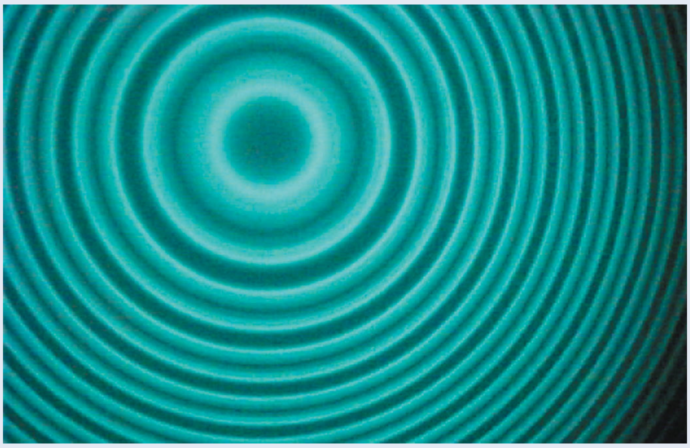
Principle:

The "Zeeman effect" is the splitting of the spectral lines of atoms within a magnetic field. The simplest is the splitting up of one spectral line into three components called "normal Zeeman effect". Usualy the phenomenon is more complex and the central line splits into many more components. This is the "anomalous Zeeman effect". Both effects can be studied using a cadmium lamp as a specimen. The cadmium lamp is submitted to different magnetic flux densities and the splitting of the red cadmium line (normal Zeeman effect) and that of a green cadmium line (anomalous Zeeman effect) is investigated using a Fabry-Perot interferometer. The evaluation of the results leads to a fairly precise value for Bohr's magneton.

What you need:

		P2511007 with Magnetic system, variable	P2511005 with Electromagnet
Fabry-Perot interferometer	09050.02	1	1
Cadmium lamp for Zeeman effect	09050.20	1	1
Electromagnet w/o pole shoes	06480.01		1
Pole pieces, drilled, conical	06480.03		1
Rot.table for heavy loads	02077.00		1
Magnetic System, variable	06327.00	1	
Power supply for spectral lamps	13662.97	1	1
Variable transformer 25 V~/20 V-, 12 A	13531.93		1
Capacitor, electrolyte, 22000 mic-F	06211.00		1
Digital multimeter 2010	07128.00		1
Optical profile-bench, l = 1000mm	08282.00	1	1
Base for optical profile-bench, adjustable	08284.00	2	2
Slide mount for optical bench, h = 30 mm	08286.01	6	5
Slide mount for optical profil-bench, h = 80 mm	08286.02	2	2
Lens holder	08012.00	4	4
Lens, mounted, f = +50 mm	08020.01	2	2
Lens, mounted, f = +300 mm	08023.01	1	1
Iris diaphragm	08045.00	1	1
Polarising filter, on stem	08610.00	1	1
Polarization specimen, mica	08664.00	1	1
Connecting cord, 32 A, l = 250 mm, red	07360.01		1
Connecting cord, 32 A, l = 250 mm, blue	07360.04		1
Connecting cord, 32 A, l = 500 mm, red	07361.01		1
Connecting cord, 32 A, l = 500 mm, blue	07361.04		1
Connecting cord, 32 A, l = 750 mm, red	07362.01		1
Connecting cord, 32 A, l = 1000 mm, red	07363.01		1
Connecting cord, 32 A, l = 1000 mm, blue	07363.04		1
CCD camera Moticam 352 for PC, 0.3 megapixels	88037.01	1	1
PC, Windows® XP or higher			

Complete Equipment Set, Manual on CD-ROM included
Zeeman effect P25110 05/07



Interference rings with the anomalous Zeeman effect.

Tasks:

1. Using the Fabry-Perot interferometer and a self made telescope the splitting up of the central lines into different lines is measured in wave numbers as a function of the magnetic flux density.
2. From the results of point 1. a value for Bohr's magneton is evaluated.
3. The light emitted within the direction of the magnetic field is qualitatively investigated.

Normal and anomalous Zeeman effect

LEP 5.1.10 -05

Related topics

Quantization of energy levels, Bohr's atomic model, vector model of atomic states, orbital angular moment, electron spin, Bohr's magneton, interference of electromagnetic waves, Fabry-Perot interferometer.

Principle

The "Zeeman effect" is the energy shift of atomic states caused by an magnetic field. This shift is due to the coupling of the electron orbital angular momentum to the external magnetic field. The normal Zeeman effect occurs when there is no spin magnetic moment – states with zero spin are necessary. In singulett systems the spins of the electrons cancel each other i.e. add up to zero. The energy shift of the atomic states in an outer magnetic field can be observed by the wavelength shift of the radiation emitted in atomic transitions between these states.
Generally there is not only a magnetic moment of the orbit of an electron state, but also a magnetic moment of the electron spin. This leads to a more complicated behaviour of the atomic states in an outer magnetic field. This is called anomalous Zeeman Effect and can be observed in atomic transitions where non-singulett states are involved.

Equipment

Fabry–Perot interferometer for 643.847 nm and 508.588 nm	09050.02	1
Cadmium lamp for Zeeman effect	09050.20	1
Electromagnet without pole pieces	06480.01	1
Pole pieces, drilled, conical	06480.03	1
Rotating table for heavy loads	02077.00	1
Power supply for spectral lamps	13662.97	1
Variable transformer, 25 V AC/20 V DC, 12 A	13531.93	1
Capacitor, electrolytic, 22000 µF	06211.00	1
Digital multimeter	07128.00	1
Optical profile-bench, l = 1000 mm	08282.00	1
Base for opt. profile-bench, adjust.	08284.00	2
Slide mount for opt. profile-bench, h = 30 mm	08286.01	5
Slide mount for opt. profile-bench, h = 80 mm	08286.02	2
Lens holder	08012.00	4
Lens, mounted, f = +50 mm	08020.01	2
Lens, mounted, f = +300 mm	08023.01	1
Iris diaphragm	08045.00	1
Polarizing filter, on stem	08610.00	1
Polarization specimen, mica	08664.00	1
Connecting cord, l = 25 cm, 32 A, red	07360.01	1
Connecting cord, l = 25 cm, 32 A, blue	07360.04	1
Connecting cord, l = 50 cm, 32 A, red	07361.01	1
Connecting cord, l = 50 cm, 32 A, blue	07361.04	1
Connecting cord, l = 75 cm, 32 A, red	07362.01	1
Connecting cord, l = 100 cm, 32 A, red	07363.01	1
Connecting cord, l = 100 cm, 32 A, blue	07363.04	1
CDC-Camera for PC incl. measurement software*	88037.00	1

PC with USB interface, Windows®98SE / Windows®Me / Windows®2000 / Windows®XP

*For classical version of the Zeeman Effect (P2511001), alternative to CCD-Camera incl. measurement software:

Sliding device, horizontal	08713.00	1
Swinging arm	08256.00	1
Plate holder with tension spring	08288.00	1
Screen, with aperture and scale	08340.00	1

Fig.1a: Experimental set-up

Tasks

1a. Normal Zeeman effect: Transversal and longitudinal observation of the splitting of the red 643.847 nm Cd-line in the magnetic field showing the normal Zeeman effect.

1b. Anomalous Zeeman effect: Transversal and longitudinal observation of the splitting of the green 508.588 nm Cd-line in the magnetic field showing the anomalous Zeeman effect.

2. Observation of the effect of polarization filter and polarization filter combined with $\lambda/4$ plate for the splitted green and red lines in transversal and longitudinal direction.

3. Measurement of the frequency shift with help of the CCD camera and the supplied measurement software or with the screen with scale and the sliding device in the classical version for both of the above mentioned spectral lines.

Set-up and Procedure

The electromagnet is put on the rotating table for heavy loads and mounted with the two pole-pieces with holes so that a gap large enough for the Cd-lamp (9-11 mm) remains. The pole-pieces have to be tightened well! The Cd-lamp is inserted into the gap without touching the pole-pieces and connected to the power supply for spectral lamps. The coils of the electromagnet are connected in parallel and via an ammeter connected to the variable power supply of up to 20 VDC,12 A. A capacitor of 22 000 µF is in parallel to the power output to smoothen the DC-voltage.

The optical bench for investigation of the line splitting carries the following elements (their approximate position in cm in brackets):

(80) CDC-Camera
(73) L_3 = +50 mm
(68) Screen with scale (only in classical version)
(45) Analyser
(39) L_2 = +300 mm
(33) Fabry-Perot Étalon
(25) L_1 = +50 mm
(20) Iris diaphragm
(20) Drilled pole-pieces
(0) Cd-spectral lamp on rotating table

Initial adjustment and observation of the longitudinal Zeeman effect is done without the iris diaphragm. During observation of the transverse Zeeman effect the iris diaphragm is illuminated by the Cd-lamp and acts as the light source. The lens L_1 and a lens of f = 100 mm, incorporated in the étalon, create a nearly parallel light beam which the Fabry-Perot étalon needs for producing a proper interference pattern.

For the observation of the normal Zeeman effect the red colour filter is to be inserted in the holder of the étalon. For the observation of the anomalous Zeeman effect the red colour filter is to be removed from the étalon and the 508 nm interference filter is to be attached onto the holder of the +300 mm lens L_2 beneath the lens (so that there are less disturbing reflections between Fabry Perot interferometer and interference filter).

The étalon produces an interference pattern of rings which can be observed through the telescope formed by L_2 and L_3. The ring diameters can be measured using the CCD-camera and the software supplied with it. In the classical version the interference pattern is produced within the plane of the screen with a scale mounted on a slide mount which can laterally be displaced with a precision of 1/100th mm. The measurement here can be done by displacing the slash representing the “0“ of the scale.

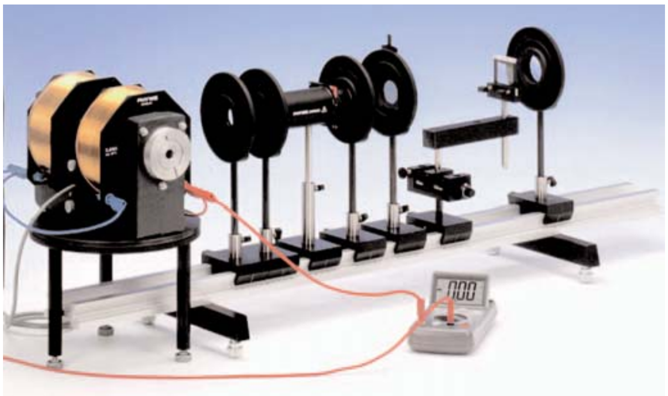


Fig.1b: Set-up for the classical version of the experiment.

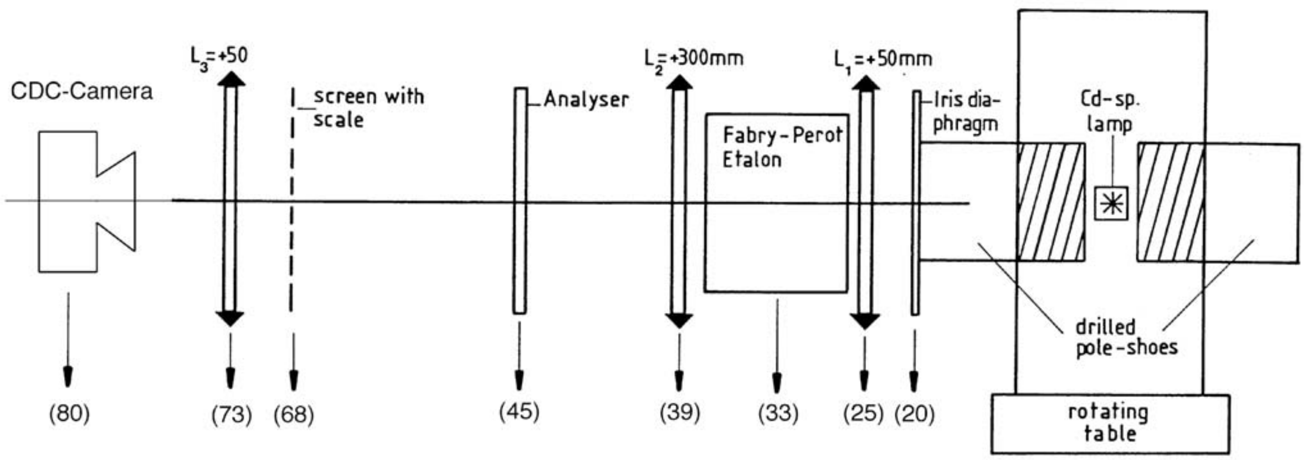


Fig. 2: Arrangement of the optical components.

Initial adjustment: The rotating table is adjusted so that the centres of the holes in the pole-pieces lie about 28 cm above the table. The optical bench with all elements (except iris diaphragm and CCD-camera) mounted, is then moved closer to the electromagnet so that one of the outlet holes of the pole-pieces coincides with the previous position of the iris diaphragm. L_1 is then adjusted so that the outlet hole is within its focal plane. All other optical elements of Fig.2. are subsequently readjusted with respect to their height.
The current of the coils is set to 5 A (increase in light intensity of the Cd-lamp!) and the ring interference pattern in axial direction is observed through L_3 by the eye. The splitting of the line should be well visible. The pattern must be centered and sharp which is eventually achieved by a last, slight movement of the étalon (to the right or to the left) and by displacement of L_2 (vertically and horizontally) and of L_3.
Finally the CCD-camera is focussed so that far away things are clear and mounted to the optical bench and adjusted in horizontal and vertical position as well as in tilt until a clear picture of the ring pattern is visible on the computer screen. For installation and use of the camera and software please refer to the manual supplied with the camera.
In the classical version the screen with scale is shifted in a way that the slash representing the „0“ of the scale is clearly seen coinciding, for instance, with the centre of the fairly bright inner ring. The scale itself must be able to move horizontally along the diameter of the ring pattern. (Set-up see Fig. 1b.)
Hint: best results are achieved when the experiment is carried out in a darkened room.
The electromagnet is now turned by 90° and the iris diaphragm is inserted for transversal observation.

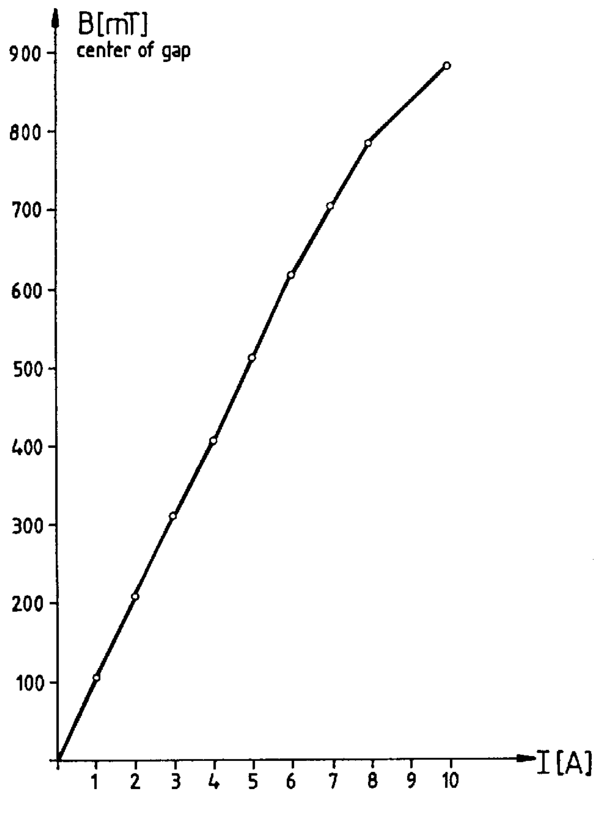


Fig. 3: Magnetic flux density B in the centre of gap without the Cd-lamp (gap width: 9 mm) as a function of the coil current.

Remark: For later evaluations the calibration curve of the magnetic flux density versus the coil current has to be recorded previously. This can be done if a teslameter is available. Otherwise the results of Fig. 3 can be used. The curve of Fig. 3 was recorded by measuring the flux density in the centre of the gap in the absence of the Cd-lamp. For the evaluations these centre-values were increased by 3.5 % to account for the non-uniform flux distribution within the gap.

Theory

As early as 1862, Faraday investigated whether the spectrum of coloured flames changes under the influence of a magnetic field, but without success. It was not until 1885 that Fievez from Belgium was able to demonstrate an effect, but it was forgotten and only rediscovered 11 years later by the Dutchman Zeeman, who studied it together with Lorentz.
Here the effect is demonstrated with the light of a Cadmium lamp and the help of a Fabry-Perot interferometer for resolving a small part of the spectrum preselected by a color filter or an interference filter so only the light of a single atomic transition line is observed. Without field the magnetic sub-levels have the same energy but with field the degeneration of the levels with different m_J is cancelled and the line is split.
Cadmium has the electron structure (Kr) $4d^{10}\ 5s^2$, i.e. the outer shell taking part in optical transitions is composed by the two $5s^2$ electrons that represent a completed electron shell. ((Kr) = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6$.) This is similar to the outer electron structure of Helium but also of Mercury. A scheme of the energy levels of Cd is shown in Fig.4. In a completed shell in it's ground state the electron spins always compensate each other – they are anti-parallel. If the total electron spin is zero, also the magnetic moment connected to electron spin is zero. Atomic states with zero total spin are called singulett states. So in transitions between different singulett states the magnetic moment of spin does not play a role, as is the case with the normal Zeeman effect. Electric dipole radiation as in common optical transitions does not change the electron spin except in heavy atoms with jj-coupling, so transitions are normally between different states in the same multiplicity system. But Fig. 4 shows there is some jj-coupling in Cadmium.
The transition used to demonstrate the normal Zeeman effect is 3 $^1D_2 \rightarrow$ 2 $^1P_1$ with 643.847 nm and the transition used to demonstrate the anomalous Zeeman effect is 2 $^3S_1 \rightarrow$ 2 $^3P_2$ with 508.588 nm.
In a term like 2 $^3S_1$ the first number "2" denotes the main quantum number of the radiating electron with respect to the atom's ground state (that is counted as "1"), here this is really the 6th s-shell since $5s^2$ is the ground state. (This is why the 2 P – states are below the 2 S – states, 2 $^3P_2$ denotes the 5th p-shell since Krypton has $4p^6$.) The upper "3" denotes the multiplicity, that is $2s$+1 with s here the spin quantum number. The lower "1" denotes the quantum number j of the total angular momentum, i.e. $j = l+s$, $l+s$-1, … , l-s with l the quantum number of the angular momentum of the orbit. "S", "P", "D", "F" denote the actual value of l, i.e. "S" means l = 0, "P" means l = 1, …
3 $^1D_2 \rightarrow$ 2 $^1P_1$ is a transition within the singulett system so the spin magnetic moments have no effect. But in the transition 2 $^3S_1 \rightarrow$ 2 $^3P_2$ triplett states are involved and the spin magnetic moment does not vanish in all sub-states.
The selection rule for optical transitions is Δm_J = 0, ±1 and the radiation belonging to transitions with Δm_J = 0 are called π-lines and the ones with Δm_J = ±1 are called σ-lines. With the magnetic field turned on in the absence of the analyser three lines can be seen simultaneously in the normal Zeeman effect

in transversal observation. In the case of the anomalous Zeeman effect three groups of three lines appear. Inserting the analyser in the normal Zeeman effect two σ-lines can be observed if the analyser is in the vertical position, while only the π-line appears if the analyser is turned into its horizontal position (transversal Zeeman effect). In the anomalous Zeeman effect there are two groups of three σ-lines in vertical polarization and one group of three π-lines in horizontal polarization. Turning the electromagnet by 90° the light coming from the spectral lamp parallel to the direction of the field (longitudinal) can also be studied trough the holes in the pole-pieces. It can be shown that this light is circular polarized light (longitudinal Zeeman effect). Fig. 5 summarizes the facts.

A λ/4-plate is generally used to convert linear into elliptical polarized light. In this experiment the λ/4-plate is used in the opposite way. With the λ/4-plate inserted before the analyser, the light of the longitudinal Zeeman effect is investigated. If the optical axis of the λ/4-plate coincides with the vertical, it is observed that some rings disappear if the analyser is at an angle of +45° with the vertical while other rings disappear for a position of –45°. That means that the light of the longitudinal Zeeman effect is polarized in a circular (opposed way). The π-lines are longitudinally not observable.

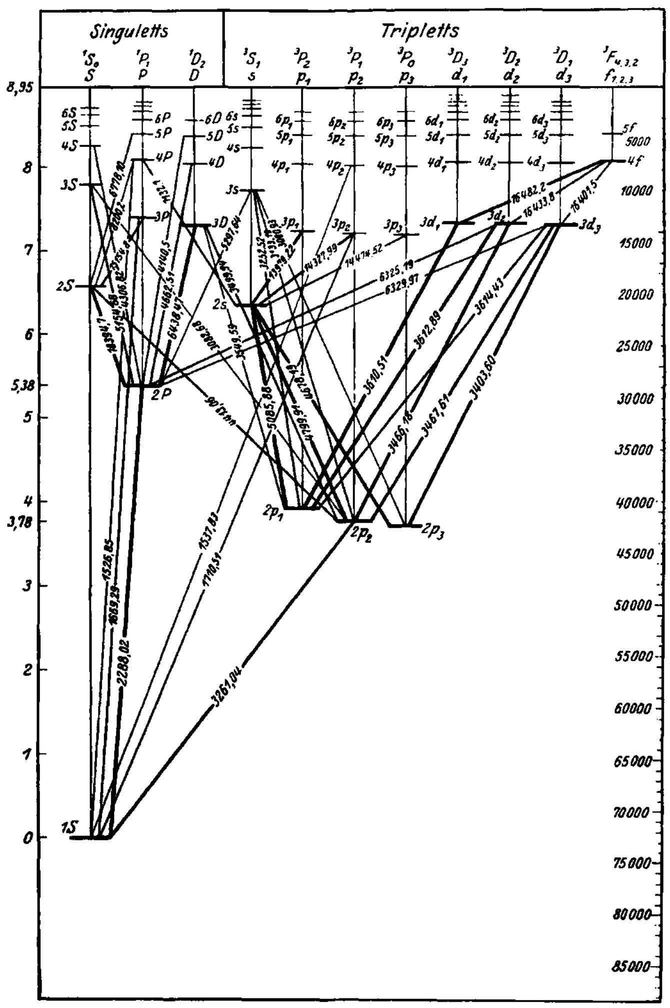


Fig. 4: The atomic states of Cadmium, wavelength in Å = 0.1 nm

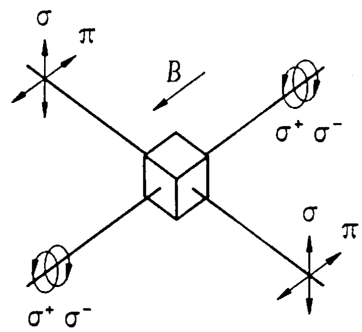


Fig. 5: Longitudinal and transversal Zeeman effect.

In the **normal Zeeman effect** with the transition 3 $^1D_2 \rightarrow 2\ ^1P_1$ with 643.847 nm the electron spins cancel each other in both the initial and final state and the energy of an atomic state in a magnetic field depends only on the magnetic moments of the electron orbit.

The magnetic moment of the orbital angular momentum $\vec{l}$ is

$$\vec{\mu}_l = -\frac{e}{2m_e}\,\vec{l} = -g_l\,\mu_B\,\frac{\vec{l}}{\hbar} \qquad (*)$$

with Bohr's magneton

$$\mu_B = \frac{e\hbar}{2m_e} = 9.274 \cdot 10^{-24}\,\mathrm{Am}^2$$

and the gyromagnetic factor of orbital angular momentum $g_l = 1$.

In the vector model of the atom the energy shifts can be calculated. It is assumed, that angular moments and magnetic moments can be handled as vectors. Angular moment and the magnetic moment connected with it are antiparallel because of the negative electron charge. The amount of the orbital magnetic moment of the orbital angular momentum $\vec{l}$, with quantum number l such that

$$|\vec{l}| = \hbar\,\sqrt{l\,(l+1)}\text{, is:}\quad \mu_l = \mu_B\,\sqrt{l\,(l+1)}$$

In case of LS-coupling (Russel-Saunders coupling, spin-orbit coupling) for many electron systems is the amount of the total angular momentum

$$|\vec{J}| = |\vec{L} + \vec{S}| = \hbar\sqrt{J(J+1)} \;\text{ with }\; \vec{S} = \sum \vec{s}_i$$

the sum of the spins of the single electrons and

$$\vec{L} = \sum \vec{l}_i$$

the sum of the orbit angular moments of the single electrons. Here it is

$$\vec{S} = 0\,.$$

So $|\vec{J}| = |\vec{L}| = \hbar\sqrt{L(L+1)}\,.$

The amount of the component of the corresponding magnetic moment $\vec{\mu}_J$ in direction of $\vec{J}$ is:

$$|(\vec{\mu}_J)_J| = |\vec{\mu}_L| = \mu_B \sqrt{L(L+1)} = g_J \mu_B \sqrt{J(J+1)}$$

with $g_J = 1$.

Observable is only the projection of the magnetic moment on $\vec{J}$

$$(\vec{\mu}_J)_J = -g_J \mu_B \frac{\vec{J}}{\hbar}$$

with it's quantization with respect to z-axis

$$(\vec{\mu}_J)_{J,z} = -m_J g_J \mu_B$$

with the magnetic quantization number m_J with $m_J = J$, J-1, …,-J
The interaction energy with the outer magnetic field B_0 along the z-axis is then

$$V_{m_J} = -m_J g_J \mu_B B_0.$$

Here the used transition for the **normal Zeeman effect** is 3 $D_2 \rightarrow 2\ ^1P_1$.
So in the initial state is $L = 2$, $S = 0$ and $J = 2$. m_J may have the values m_J = -2, -1, 0, 1, 2. The gyromagnetic factor is $g_i = 1$ and the energy difference between two neighbouring sub-states of the initial state is then $\Delta E = -1\mu_B B_0$.
In the final state is $L = 1$, $S = 0$ and $J = 1$. m_J may have the values m_J = -1, 0, 1. The gyromagnetic factor is $g_f = 1$ and the energy difference between two neighbouring sub-states of the final state is then $\Delta E = -1\mu_B B_0$, too, i.e. for transitions with the same Δm_J between initial and final state the energy shift is for initial and final state the same – so they have altogether the same frequency.
Fig. 6 shows the resulting transition diagram.
For electrical dipole transitions the selection rule states Δm_J = 1, 0, -1.
The energy shift of a transition between initial state with m_{Ji} and g_{J_i} and final state with m_{J_f} and g_f is then

$$V_{m_{J_i}} - V_{m_{J_f}} = (m_{J_f} g_f - m_{J_i} g_i)\, \mu_B B_0$$

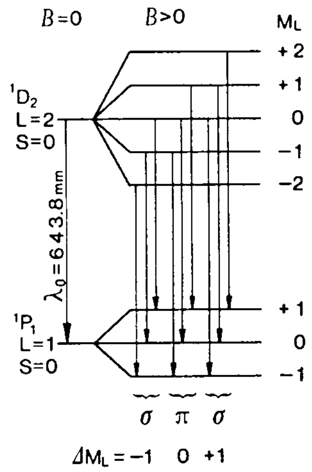


Fig. 6: Energy shift of the atomic states

and here the $(m_{J_f} g_f - m_{J_i} g_i)$-values are simply equal to Δm_J. So in case of LS-coupling in the normal Zeeman effect three equidistant lines are expected in this transition with a distance in frequency or wave number proportional to the magnetic field strength. The polarization of the transitions with $\Delta m_J = 0$ in transversal observation is parallel to the magnetic field (here horizontal) and of the other transitions the polarization is perpendicular to that.

The **anomalous Zeeman effect** is the more general case where the electron spins do not cancel each other and the energy of an atomic state in a magnetic field depends on both the magnetic moments of electron orbit and electron spin.
The magnetic moment of the orbital angular momentum $\vec{l}$ is as above (see (*)) and the magnetic moment of the spin $\vec{s}$ is

$$\vec{\mu}_s = -\frac{e}{2m_e}\vec{s} = -g_s \mu_B \frac{\vec{s}}{\hbar}$$

with the gyromagnetic factor of orbital angular momentum $g_s = 2.0023$.
Additional to the orbital magnetic moment of the orbital angular momentum $\vec{l}$ the amount of the spin magnetic moment of the spin $\vec{s}$, with quantum number s such that

$$|\vec{s}| = \hbar\sqrt{s\,(s+1)}\,,$$

has to be taken into account:

$$\mu_s = |-g_s \mu_B \sqrt{s\,(s+1)}|$$

In case of LS-coupling (Russel-Saunders coupling, spin-orbit coupling) for many electron systems the amount of the total angular momentum is

$$|\vec{J}| = |\vec{L} + \vec{S}| = \hbar\sqrt{J(J+1)} \text{ with } \vec{S} = \sum \vec{s}_i$$

the sum of the spins of the single electrons and

$$\vec{L} = \sum \vec{l}_i$$

the sum of the orbit angular moments of the single electrons.
In the vector model it is assumed, that angular moments and both spin and orbital magnetic moments can be handled as vectors. So the cosine rule applies for the sum of two vectors with an angle between them. The amount of the component of the corresponding magnetic moment $\vec{\mu}_J$ in direction of $\vec{J}$ is with the approximation $g_s \approx 2$:

$$|(\vec{\mu}_J)_J| = |\vec{\mu}_L| \cos(\vec{L},\vec{J}) + |\vec{\mu}_S| \cos(\vec{S},\vec{J})$$

$$= \mu_B\left(\sqrt{L\,(L+1)}\cos(\vec{L},\vec{J}) + 2\sqrt{S\,(S+1)}\cos(\vec{S},\vec{J})\right)$$

$$|(\vec{\mu}_J)_J| = \frac{3J(J+1)+S(S+1)-L(L+1)}{2\sqrt{J(J+1)}}\mu_B$$

$$= g_J \mu_B \sqrt{J(J+1}$$

with

$$g_1 = 1 + \frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)}\quad.$$

Observable is only the projection of the magnetic moment on $\vec{J}$

$$(\vec{\mu}_J)_J = -g_J \mu_B \frac{\vec{J}}{\hbar}$$

with it's quantization with respect to z-axis

$$(\vec{\mu}_J)_{J,z} = -m_J g_J \mu_B$$

with the magnetic quantization number m_J with $m_J = J$, J-1, …,-J

The interaction energy with the outer magnetic field B_0 along the z-axis is then

$$V_{m_J} = -m_J g_J \mu_B B_0.$$

Here for the **anomalous Zeeman effect** the used transition is is $2\,^3S_1 \rightarrow 2\,^3P_2$.

So in the initial state is L = 0, S = 1/2 + 1/2 = 1 and J = 1 + 0 = 1. m_J may have the values m_J = -1, 0, 1. The gyromagnetic factor is

$$g_i = 1 + \frac{1(1+1)+(1+1)-0(0+1)}{2 \cdot 1(1+1)} = 2$$

and the energy difference between neighbouring sub-states of the initial state is then

$$\Delta E = -2\mu_B B_0.$$

In the final state is L = 1, S = 1 and J = 2. m_J may have the values m_J = -2, -1, 0, 1, 2. The gyromagnetic factor is

$$g_f = 1 + \frac{2(2+1)+(1+1)-1(1+1)}{2 \cdot 2(2+1)} = \frac{3}{2}$$

and the energy difference between neighboured sub-states of the final state is then

$$\Delta E = -\frac{3}{2}\mu_B B_0.$$

Fig. 7 shows the resulting transition diagram.

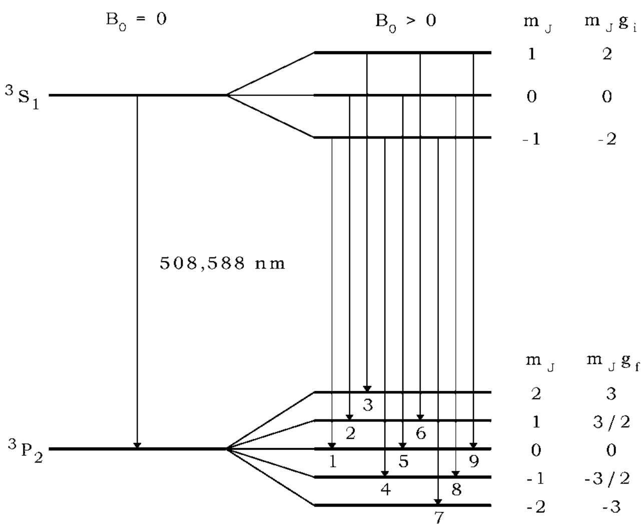


Fig. 7: Energy shift of the atomic states

For electrical dipole transitions the selection rule states Δm_J = 1, 0, -1.

The energy shift of a transition between initial state with m_{J_i} and g_{J_i} and final state with m_{J_f} and g_f is then

$$V_{m_{J_i}} - V_{m_{J_f}} = (m_{J_f} g_f - m_{J_i} g_i)\, \mu_B B_0$$

The following table shows the energy shifts of the transitions:

No.	1	2	3	4	5	6	7	8	9
Δm_j	1	1	1	0	0	0	-1	-1	-1
$m_{J_f} g_f - m_{J_i} g_i$	-2	-3/2	-1	-1/2	0	1/2	1	3/2	2

So in case of LS-coupling in the anomalous Zeeman effect nine equidistant lines are expected in this transition instead of three without spin magnetism. The polarization of the transitions with Δm_J = 0 in transversal observation is parallel to the magnetic field (here horizontal) and the polarization of the other transitions is perpendicular to the magnetic field.

At observing the σ-lines of the transversal Zeeman effect it is easy to see that the amount of splitting increases with increasing magnetic field strength. For a quantitative measurement of this splitting in terms of number of wavelengths, a Fabry-Perot interferometer is used, the functioning of which has to be explained:

The Fabry-Perot étalon has a resolution of approximately 400000. That means that a wavelength change of less then 0.002 nm can still be detected.

The étalon consists of a quartz glass plate of 3 mm thickness coated on both sides with a partially reflecting layer (90 % reflection, 10 % transmission). Let us consider the two partially transmitting surfaces (1) and (2) in Fig.8 seperated by a distance t. An incoming ray forming an angle with the plate normal will be split into the rays AB, CD, EF, etc. the path difference between the wave fronts of two adjacent rays (e.g. AB and CD) is

$$\delta = \mu \cdot (\mathrm{BC} + \mathrm{CK})$$

where BK is defined normal to CD and μ is the refractive index of quartz at 509 nm, μ = 1.4519. At 644 nm is μ = 1.4560. With

$$\mathrm{CK} = \mathrm{BC}\cos 2\theta$$

and

$$\mathrm{BC} \cdot \cos\theta = t$$

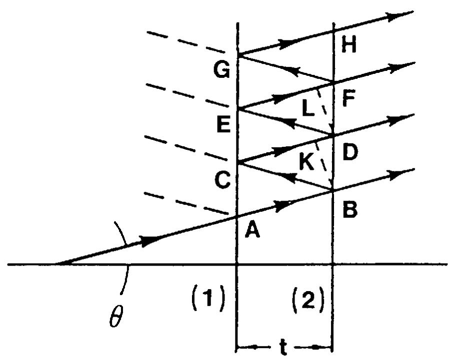


Fig. 8: Reflected and transmitted rays at the parallel surfaces (1) and (2) of the étalon. The étalon spacing is t = 3 mm.

we obtain

$$\delta = \mu \cdot \mathrm{BCK} = \mu \cdot \mathrm{BC}(1 + \cos 2\theta)$$
$$= 2\mu \cdot \mathrm{BC} \cos^2\theta = 2\mu \cdot t \cdot \cos\theta$$

and for a constructive interference it is:

$$n\lambda = 2\mu \cdot t \cdot \cos\theta \qquad (1)$$

where n is an integer and λ the light's wavelength. Equation (1) is the basic interferometer equation. Let the parallel rays B, D, F, etc. be brought to a focus by the use of a lens of focal length f as shown in Fig. 9.

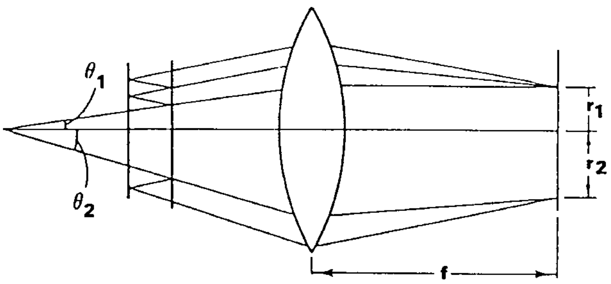


Fig. 9: Focusing of the light emerging from a Fabry-Perot étalon. Light entering the étalon at an angle θ is focused onto a ring of radius $r = f\theta$ where f is the focal length of the lens.

For θ fulfilling equation (1) bright rings will appear in the focal plane with the radius

$$r_n = f \tan \theta_n \approx f\theta_n \qquad (2)$$

for small values θ_n, e.g. rays nearly parallel to the optical axis. Since

$$n = \frac{2\mu \cdot t}{\lambda} \cos \theta_n = n_0 \cos \theta_n = n_0\left(1 - 2\sin^2 \frac{\theta_n}{2}\right)$$

with

$$n_0 = \frac{2\mu \cdot t}{\lambda}$$

we finally obtain

$$n = n_0 \left(1 - \frac{\theta_n^2}{2}\right)$$

or

$$\theta_n = \sqrt{\frac{2(n_0 - n)}{n_0}} \qquad (3)$$

If θ_n corresponds to a bright fringe, n is an integer. However is n_0 the interference condition for the center (for $\theta = 0$) generally not an integer.
If n_1 is the interference order of the first ring, it is $n_1 < n_0$ since $n_1 = n_0 \cos \theta_n$. We then let

$$n_1 = n_0 - \varepsilon \,;\; 0 < \varepsilon < 1$$

where n_1 is the closest integer to n_0 (smaller than n_0). In general is for the p^{th} ring of the pattern, measured starting from the center, the following is valid:

$$n_p = (n_0 - \varepsilon) - (n_p - 1) \qquad (4)$$

Combining equation (4) with equations (2) and (3), we obtain for the radii of the rings, writing r_p for r_{n_p},

$$r_p = \sqrt{\frac{2f^2}{n_0}} \cdot \sqrt{(p-1) + \varepsilon} \qquad (5)$$

We note that the difference between the squares of the radii of adjacent rings is a constant:

$$r_{p+1}^2 - r_p^2 = \frac{2f^2}{n_0} \qquad (6)$$

ε can be determined graphically plotting $r_p{}^2$ versus p and extrapolating to $r_p{}^2 = 0$.
Now, if there are two components of a spectral line (splitting of one central line into two components) with wavelengths λ_a and λ_b, which are very close to one another, they will have fractional orders at the center ε_a and ε_b:

$$\varepsilon_a = \frac{2\mu \cdot t}{\lambda_a} = -n_{1,a} = 2\mu \cdot t \cdot k_a - n_{1,a}$$

$$\varepsilon_b = \frac{2\mu \cdot t}{\lambda_b} = -n_{1,b} = 2\mu \cdot t \cdot k_b - n_{1,b}$$

where k_a and k_b are the corresponding wave numbers and $n_{1,a}$, $n_{1,b}$ is the interference order of the first ring. Hence, if the rings do not overlap by a whole order so $n_{1,a} = n_{1,b}$ and the difference in wave numbers between the two components is

$$\Delta k = k_a - k_b = \frac{\varepsilon_a - \varepsilon_b}{2\mu \cdot t} \qquad (7)$$

Using equations (5) and (6), we get

$$\frac{r_{p+1,a}^2}{r_{p+1}^2 - r_p^2} - p = \varepsilon \qquad (8)$$

Applying equation (8) to the components a and b, yields

$$\frac{r_{p+1,a}^2}{r_{p+1,a}^2 - r_{p,a}^2} - p = \varepsilon_a$$

and

$$\frac{r_{p+1,b}^2}{r_{p+1,b}^2 - r_{p,b}^2} - p = \varepsilon_b$$

By substituting these fractional orders into equation (7), we get for the difference of the wave numbers:

$$\Delta k = \frac{1}{2\mu \cdot t}\left(\frac{r_{p+1,a}^2}{r_{p+1,b}^2 - r_p^2} - \frac{r_{p+1,b}^2}{r_{p+1,b}^2 - r_{p,b}^2}\right) \qquad (9)$$

From equation (6) we get the difference between the squares of the radii of component a:

$$\Delta_a^{p+1,p} = r_{p+1,a}^2 - r_{p,a}^2 = \frac{2f^2}{n_{0,a}}$$

this is equal to (within a very small part) the same difference for component b

$$\Delta_b^{p+1,p} = r_{p+1,b}^2 - r_{p,b}^2 = \frac{2f^2}{n_{0,b}}$$

Hence we assume

$$\Delta_a^{p+1,p} = \Delta_b^{p+1,p}$$

for all values of p. Similarly, all values

$$\delta_{a,b}^p = r_{p+1,a}^2 - r_{p+1,b}^2$$

must be equal, regardless of p (the order of interference) and their average may be taken as may be done for the different Δ-values. With δ (the difference of squares of radii of different lines of the same order of interference) and Δ (difference of squares of radii of different orders) as average values we get for the difference of the wave numbers of the components a and b:

$$\Delta k = \frac{1}{2\mu \cdot t} \cdot \frac{\delta}{\Delta} \qquad (10)$$

Note: Equation (10) shows that Δk does not depend on the dimensions used in measuring the radii of the ring system.

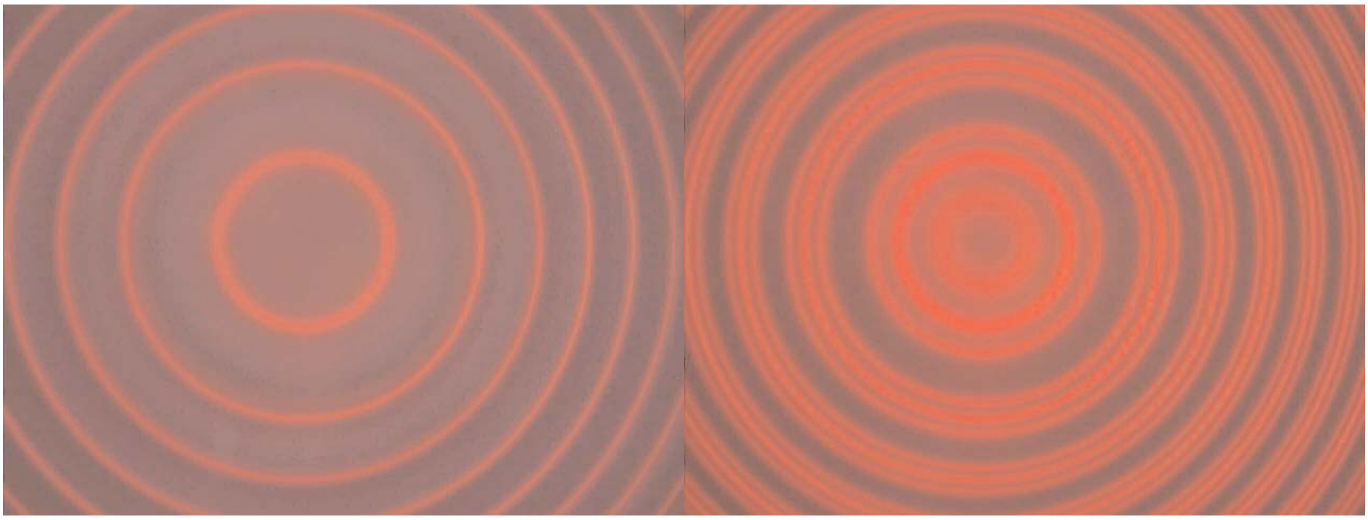


Fig.10: Normal Zeeman effect: Interference pattern without polarization filter for no coil current and for 5 A coil current. On the left there is one ring per order of interference, on the right there are three rings per order of interference

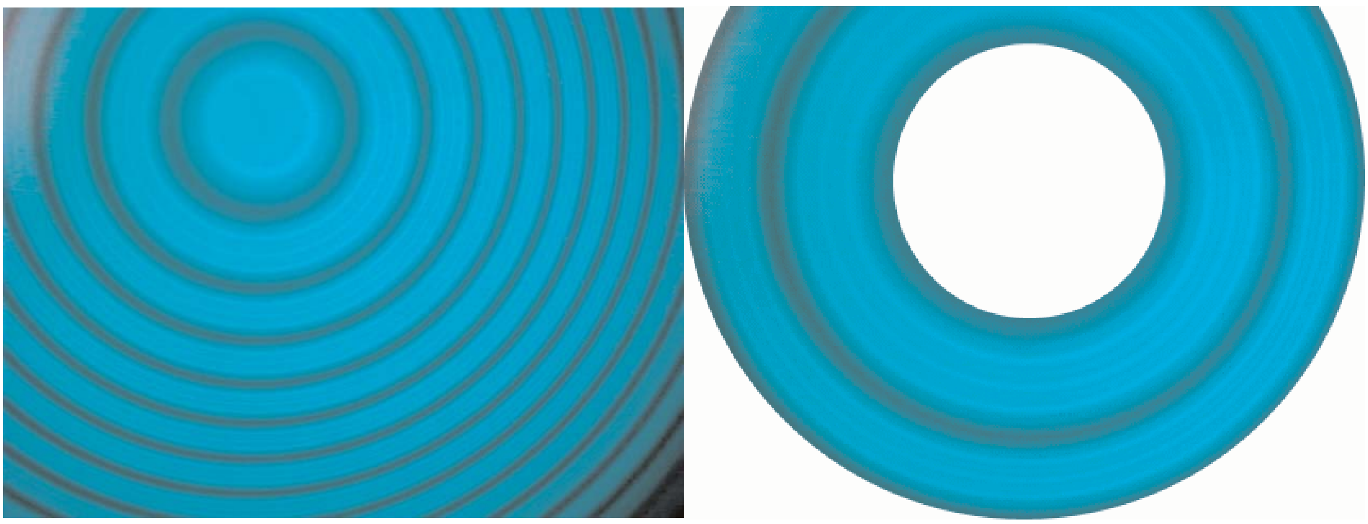


Fig.11: Anomalous Zeeman effect: Interference pattern without polarisation filter and magnified cut-out of the first completely visible two orders of interference