Duane-Hunt's displacement law; h - to be determined

- Recording the braking spectrum $N = f(\Theta)$ of the X-rays of a molybdenum anode for various high voltages U_A according to Bragg's rotating crystal method with a counter tube goniometer.
  N: Counting rate
  Θ: Angle between the primary beam and the grating plate of a monocrystal; Bragg angle.
- Determination of the angle Θ_{min} from the graph of the measured curves $N(U_A) = f(\Theta)$ (Θ_{min}: angle at which the braking spectrums first occur).
- Determination of the corresponding critical wavelength λ_{min} from Bragg's reflection condition
  $\lambda_{min} = 2\ d\ \sin\ \theta_{min}$
  d: Lattice plane distance.
- Exploring the dependency $U_A = f(\lambda_{min})$
- Determining Planck's action quantum
  $h = \frac{e}{c}\ U_A \cdot \lambda_{min}$
  e = Elementary charge
  c = Speed of light

With decreasingly high voltages, the braking spectrum of the X-rays displaces itself towards larger wavelengths. Duane and Hunt found the proportionality between the voltage U_A and the frequency of the short wave boundary ν_g in 1915:

$$U_A \sim \nu_g \text{ bzw. } \lambda_{min} \sim \frac{1}{U_A} \tag{1}$$

This proportionality can be explained only by the quantum theory with Einstein's equation

$$h \cdot \nu = e \cdot U_A \tag{2}$$

The energy $h \cdot \nu$ of the emitted X-ray quantum can maximally be equal to the kinetic energy of the fastest electrons in the X-ray tube, which is

$$W_{max} = e \cdot U_A\ . \tag{3}$$

From $h \cdot \nu = e \cdot U_A$, it follows that:

$$\lambda_{min} = \frac{h \cdot c}{e} \cdot \frac{1}{U_A} \tag{4}$$

When $h = 6.66256 \cdot 10^{-34}$ Js
$c = 2.9979 \cdot 10^{8}\ m\ s^{-1}$
and $e = 1.6021 \cdot 10^{-19}$ C
we calculate the wavelength λ_{min} of the shortwave boundary of the braking spectrum as:

$$\lambda_{min} = 1{,}2398 \cdot 10^{-6}\ V \cdot m \cdot \frac{1}{U_A}\ . \tag{5}$$

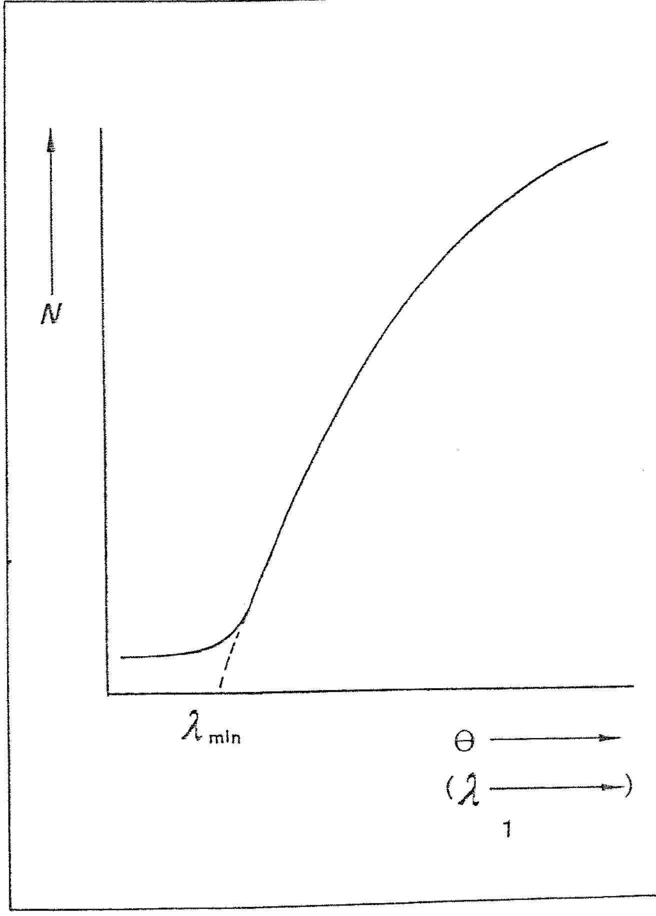

Fig. 1: Graphic determination of the shortwave boundary Θ_{min} (or λ_{min}) by extrapolating measured curve $N = f(\Theta)$.

The Duane-Hunt law is proven by the experimental determination of λ_{min}. Also Planck's action quantum can be determined with the same measuring values according to the equation

$$h = \frac{e}{c} \cdot U_A \cdot \lambda_{min} \quad (6)$$

In the experiment, the critical wavelength λ_{min} is determined as a function of U_A. X-ray spectra at various high voltages U_A are recorded with Bragg's configuration. The corresponding boundary angle is determined at which radiation starts.

The braking spectrum suddenly stops at θ_{min}; but this is concealed during measurement by apparatus influences. Therefore the short wave boundary must be found by extrapolating the edge of the graph spectra as shown in Fig.1. The boundary wavelength λ_{min} then results from Bragg's reflection condition

$$\lambda_{min} = 2\, d \sin \theta_{min} \quad (7)$$

Apparatus:

1 X-ray apparatus, 42 kV, with counter, angle graduation, aperture slot collimator and holder for samples with table	554 90
1 X-ray tube	554 94
1 Sodium flouride monocrystal	554 78
alternatively	
1 Lithium fluoride monocrystal	554 77
1 End-window counter for β, γ and X-rays	559 05
1 Cable for counter tube, 1 m length	559 07
1 Digital counter	575 50
1 Ratemeter	575 52
1 Interchangeable scale demonstration meter, basic unit, measuring range 30 V∿	530 50
Power supply unit, plug-in 9.2 V	530 88
module, pass scale 30 V/100 V	530 58
2 Connecting leads, 50 cm, black	501 28

Setting up:

Pay attention to the information given in the operating instructions of the X-ray apparatus (554 90 ff).

Set up as shown in Fig. 2 and Fig. 3 (take the sequence 3.1 - 3.5 into consideration).

Release the couplings by turning the knurled screws (d) between the hinge pins for the crystal table and the holder for the counter. Set both pointers of the counter goniometer so that the tips of the pointers are exactly on top of the zero and the angle graduation (pay attention to the parallax); couple the two pins by tightening the knurled screws (d).

Important: You will only obtain satisfactory results if the setup is adjusted exactly.

Voltage on the counter: approx. 460 V (control (k))

Adjustments on the digital counter:
Frequency measurement: measuring time 100 s
Sensitivity: >1.5 V_{pp}

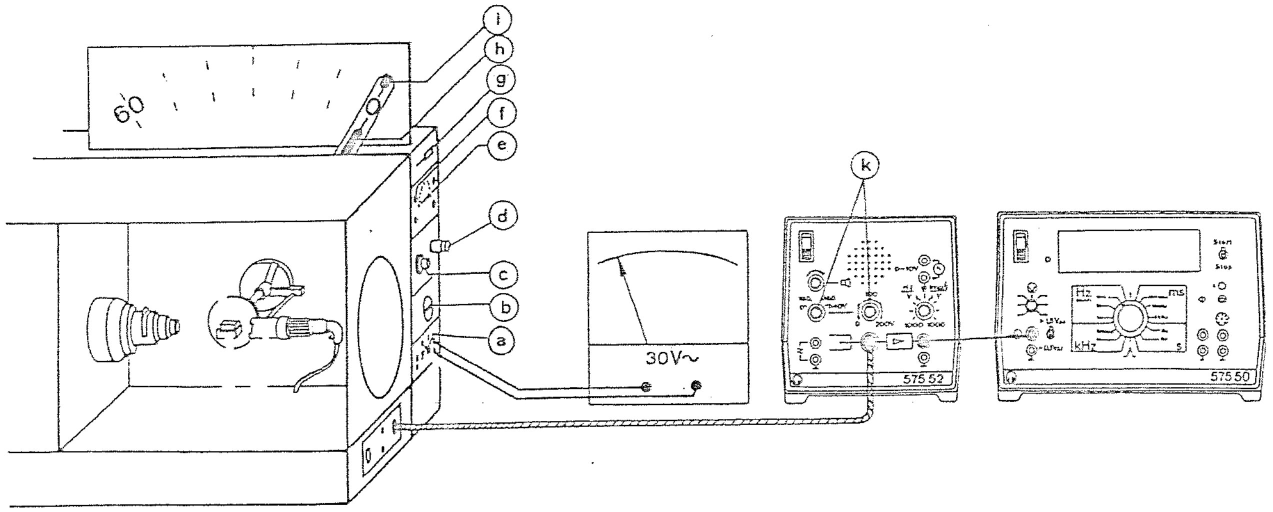

Fig. 2: Setup to record the braking spectrum as a function of high voltage

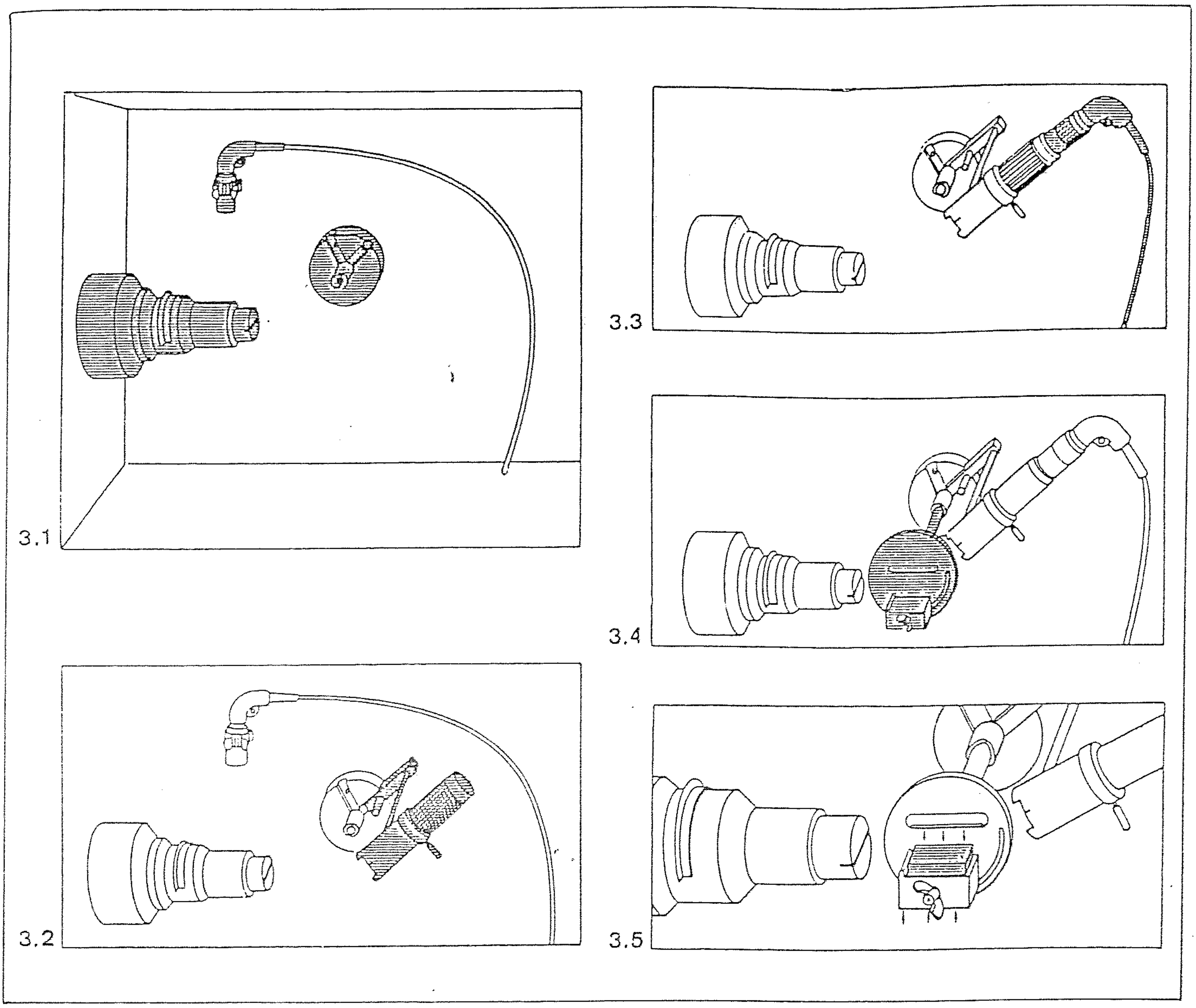

Fig. 3.1.3.5.: Installing the apparatus for Bragg's method, with the counter (559 05) in the experimentation chamber of the X-ray apparatus.

Carrying out the experiment:

1. Turn on the X-ray apparatus with switch (a); choose an operating time of > 1 h on the clock (b); turn on the high voltage U_A at setting 1 of the multiple step switch (e) on the probe (f). Set the high voltage to position 8 with the multiple step switch (e) and then set the emission current I_{EM} to 1 mA with the lever (g).

2. Read the voltage U, which is proportional to the high voltage U_A, on the demonstration meter

$(U_A = \sqrt{2} \cdot 10^3 \cdot U)$.

With adjusting knob (c), set the rotating crystal arrangement to the "crystal angle" Θ = 2.5° (pointer (h)) and the "counter tube angle" 2Θ = 5° (pointer (i)) coupled to it.

Measure the number n of impulses in 100 s.

3. Increase the angle Θ step by step from 0.5° to 6.5°, and measure the number n of impulses in 100 s every time.

4. With the switch (e), lower the high voltage step by step until it reaches position 2. Determine each time, as described in experiment parts 2 and 3, the high voltage U_A and record the braking spectrum $n = f(\Theta)$.

Measuring example:

Crystal: NaCl: 2 d = 563.94 mpm

I_{EM} = 1 mA

Angle Θ in degrees		2,5	3,0	3,5	4,0	4,5	5,0	5,5	6,0	6,5
Counting rate	U_A = 41.72 kV_s	25,21	15,70	133,2	285,1	348,87	358,96	352,14	325.05	524,95
N in	U_A = 36.77 kV_{sz}	24,37	8,49	15,46	117,02	224,82	264,89	279,62	264,48	402,21
Imp. s^{-1}	U_A = 34.65 kV_s	20,1	5,20	5.56	26,1	111,99	170,32	200,23	200,73	313,15

Table 1

The measured values N = f(Θ) for the high voltage levels 5, 4, 3, and 2 (Fig. 4), used for further evaluation, are not reproduced here.

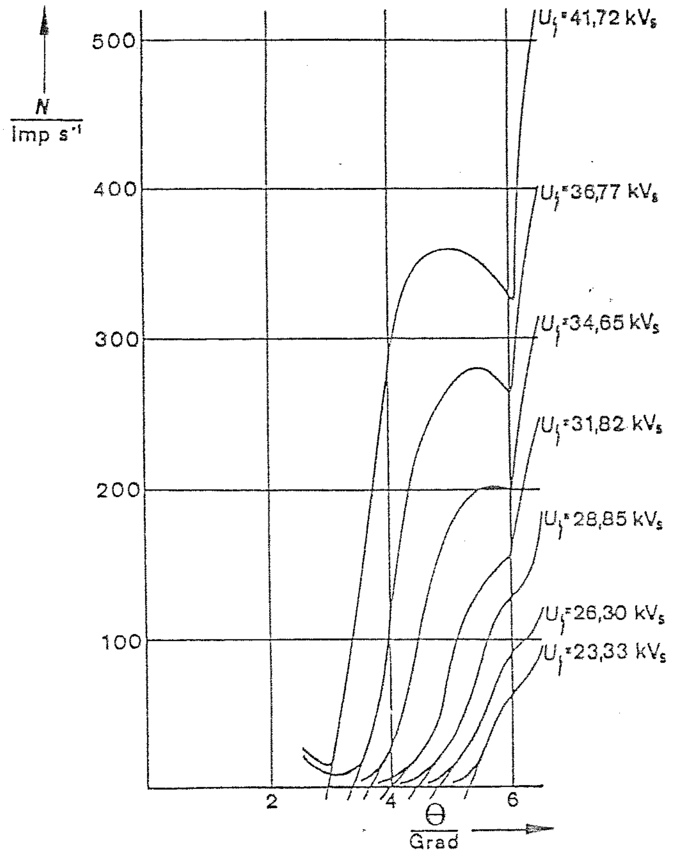

Figure 4: Braking spectra N = f(Θ) for the high voltages U_A determined by the levels 2 and 8.

Evaluation and results:

Table 2

U_A level	./.	8	7	6	5	4	3	2
U_A	kV_s	41,72	36,77	34,65	31,8	28,85	26,30	23,33
Θ_{min}	Grad	2,95	3,35	3,60	3,92	4,42	4,77	5,30
λ_{min} from (7)	pm	29,0	32,9	35,4	38,6	43,5	46,9	52,7
$\frac{1}{U_A}$	kV_s^{-1}	0,0240	0,0272	0,0289	0,0314	0,0347	0,0380	0,0429
h from (6)	10^{-34} Js	6,465	6,464	6,555	6,563	6,700	6,591	6,570

1. The displacement of the braking spectrum and the shortwave boundary with decreasing high voltages can be recognized in Fig. 4.

Graphical extrapolation gives us the values for the critical wavelength and from this λ_{min} is calculated according to Bragg's equation

$\lambda = 2\ d\ \sin\theta$ (7)

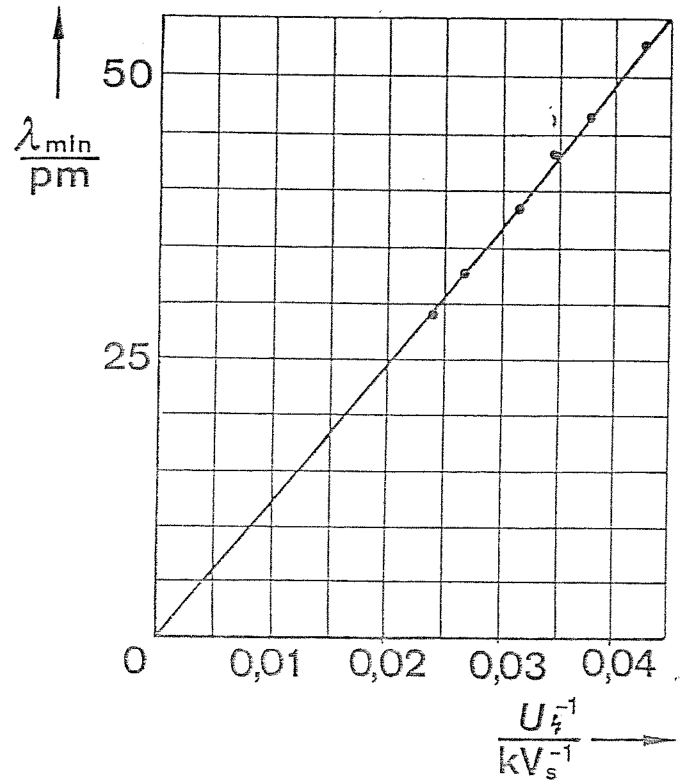

Fig. 5: $\lambda_{min} = f(\frac{1}{U_A})$.

2. The graphical representation of the wavelength, λ_{min} plotted against the reciprocal value of the voltage U_A, gives us a straight line through the origin of the coordinate system (Fig. 5).

Thus, the Duane-Hunt law is confirmed:

$$\lambda_{min} \sim \frac{1}{U_A} \quad . \qquad (1)$$

3. Additionally, the slope of the straight line, calculated from the values λ_{min} = 55 pm and U_A^{-1} = 0.045 kV_s^{-1}, gives us a proportionality factor k = 1.22 · 10^{-6} Vm. This is in agreement with the theoretically calculated value of 1.2398·10^{-6} Vm (from (5)).

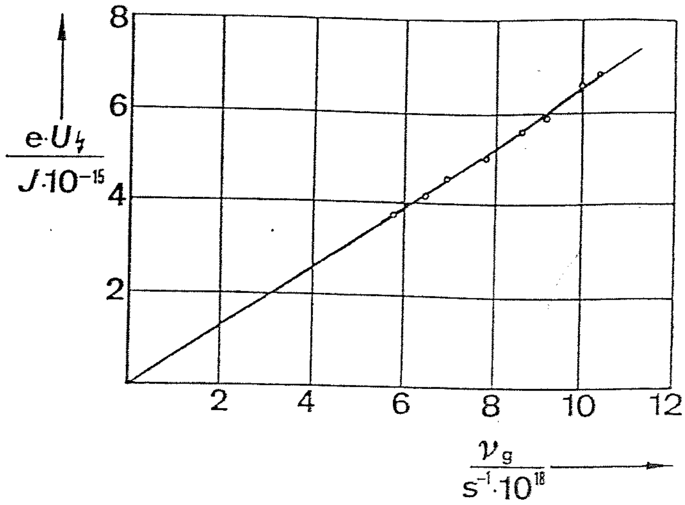

Fig. 6: $e \cdot U_A = f(\nu_g)$

4. In Fig. 6, the energy value $e\ U_A$ calculated from U_A plotted against the boundary frequencies ν_g in a graph.

Planck's action quantum can be determined both by calculation and also graphically.

a) From the equation

$$h = \frac{e \cdot \lambda_{min} \cdot U_A}{c} \qquad (6)$$

we optain the average h = 6.56 · 10^{-34} Js from the values given in table 2 of the measuring example.

b) From the slope of the line in Fig. 6, we find $h \approx 6.5 \cdot 10^{-34}$ Js with the set h of the line $e \cdot U_A = h \cdot \nu_g$.

Table 3

λ_{min}	pm	29,0	32,9	35,4	38,6	43,5	46,9	52,7
$\nu_g = \frac{c}{\lambda_{min}}$	$10^{18} s^{-1}$	10,34	9,12	8,47	7,77	6,90	6,40	5,69
$e \cdot U_A$	10^{-15} J	6,72	5,91	5,57	5,11	4,63	4,22	3,75