Chapter 6

Interaction of Gamma Quanta With Matter

Chapter Outline

6.1 Summary of the Effects of Gamma Quanta Interaction 137
6.2 Photoeffect 138
6.3 Gamma Quanta Scattering by Free Electrons (The Thomson Scattering) 140
6.4 The Compton Effect 141
6.4.1 Introduction 141
6.4.2 Laws of Conservation of Energy and Momentum 141
6.4.3 Differential Cross Sections of the Compton Effect 142
6.4.4 Total Cross Sections of the Compton Effect 145
6.4.5 Scattering by the Bound Electrons 145
6.5 Scattering by Atoms (Rayleigh Coherent Scattering) 146
6.6 Electron–Positron Pair Production 147
6.7 Total Absorption of Gamma Radiation 148
6.8 Build-Up Factor 150
6.9 Distribution of Absorbed Energy in Matter 150
References 150

6.1 SUMMARY OF THE EFFECTS OF GAMMA QUANTA INTERACTION

When gamma quanta pass through a substance, they can interact with electrons of atomic shells, nuclei, or atoms, as a whole. We distinguish two possible types of interaction:

absorption, when gamma quantum completely gives up its energy;
scattering: elastic and inelastic.

Absorption processes:

photoeffect occurs on the electrons of atomic shells;
production of electron–positron pairs occurs on nuclei and on electrons;
photonuclear reactions, reactions of the type (γ, n), (γ, p), (γ, f), etc., occur on nuclei.

Scattering processes:

elastic scattering by atoms—Rayleigh coherent scattering;
elastic scattering on nuclei—coherent nuclear scattering (resonant nuclear scattering);
the Delbruck scattering—virtual production of electron–positron pairs in the Coulomb field of the nucleus;
inelastic scattering by electrons—the Compton effect;
inelastic scattering by nuclei—incoherent nuclear scattering.

The Compton effect is called inelastic scattering. In this regard, a certain remark has to be made. In this process, gamma quantum exchanges energy with an electron, no new species are created, and kinetic energy is conserved, and that is the mark of an elastic collision. Nevertheless, in nuclear physics, this process is called inelastic to distinguish it from the scattering of gamma quanta on an atom without loss of energy, i.e., from the Rayleigh scattering.

In the processes in which gamma quantum energy is transferred to the substance, three effects are dominant: the photoelectric effect, the Compton effect, and the formation of electron–positron pairs. The probabilities of the other processes are much smaller. The relative role of each of the effects depends on the energy of the incident photons and the atomic number of the bombarded atoms, as shown in Fig. 6.1.

Radiation. https://doi.org/10.1016/B978-0-444-63979-0.00006-9

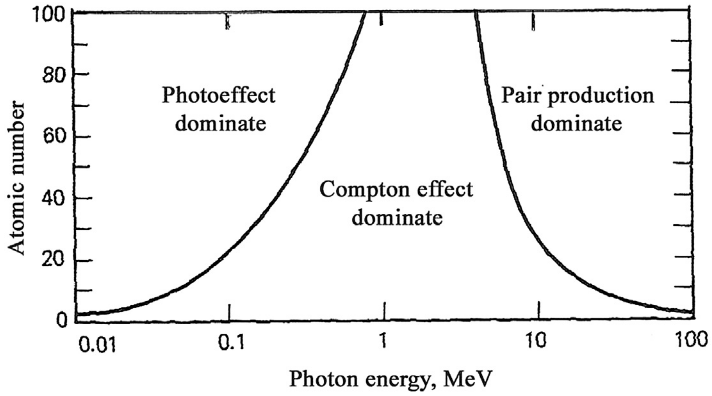

FIGURE 6.1 Curves of the cross section equality of the three main forms of photon interaction with matter. The left curve represents the region where the atomic coefficients for the photoelectric effect and Compton effect are equal, and the right curve is for the region where the atomic Compton coefficient equals the atomic pair production coefficient.

The Rayleigh scattering is also essential for the passage of gamma quanta through matter. In this process, gamma quanta retain their energy but change their direction of motion.

The main characteristic feature of the passage of charged particles through matter is the gradual nature of energy losses in small portions.

Unlike charged particles, gamma quanta experience rare collisions, in which they can lose either all (absorption) or a significant fraction of their energy (scattering). In these collisions, charged particles are born and they, in particular, act on the substance.

Charged particles gradually lose energy and have a finite range in matter. Gamma quanta collide with atoms randomly, and the radiation flux decreases by an exponential law. Thus, formally, the flux of neutral radiations reaches zero at infinity.

The interaction cross sections discussed in the following sections can refer to a single electron or to an atom as a whole. Accordingly, they are denoted by the total or differential cross sections per electron ${}_e\sigma$ or $d_e\sigma$ and by the same per atom ${}_a\sigma$ or $d_a\sigma$.

6.2 PHOTOEFFECT

A photoelectric effect is the absorption of a gamma quantum by an electron, in which the gamma quantum gives up almost its entire energy to the electron. Absorption by a free electron is impossible, as the law of momentum conservation is not satisfied.

Therefore, the photoelectric effect occurs on bound atomic electrons, and the higher the probability of the process, the stronger is the electron bound. The photoelectric effect on the K-shell nearest to the nucleus has the greatest probability, the smaller probability on the L-shell, and even smaller probability on the M-shell. The ratio of the cross sections on the K-, L-, and M-shells is

$$ {}_a\sigma_K / {}_a\sigma_L \sim 5, \quad {}_a\sigma_L / {}_a\sigma_M \sim 4. \tag{6.1}$$

Strict theoretical formulas for calculating the cross sections of the photoelectric effect of any energy quanta on atoms with any Z do not currently exist. The main theoretical calculations have an approximate character and a limited field of application. Therefore, while the expressions for the cross sections in the literature are not available, we only note the character of the cross section dependence on the atomic number of the substance and on the energy of the quanta:

$$ {}_a\sigma_\phi \sim Z^5 / h\nu \quad \text{at} \quad h\nu \gg I_K; \quad {}_a\sigma_\phi \sim Z^5 / h\nu^{3,5} \quad \text{at} \quad h\nu > I_K. \tag{6.2}$$

The dependence of the photoabsorption cross section on the energy of gamma quanta has the form of a decreasing stepped curve; each step is associated with the contribution of one of the inner shells of the atom. In intervals between steps, the cross section varies monotonically, and in double logarithmic coordinates, it is practically linear. The dependence of the photoelectric absorption coefficient on the gamma quantum energy is shown in Fig. 6.2.

It is seen that, starting at high energies, the cross section increases with decreasing energy, until the quantum energy reaches the binding energy of the electron on the K-shell. With further decrease of energy, the photoelectric effect on the

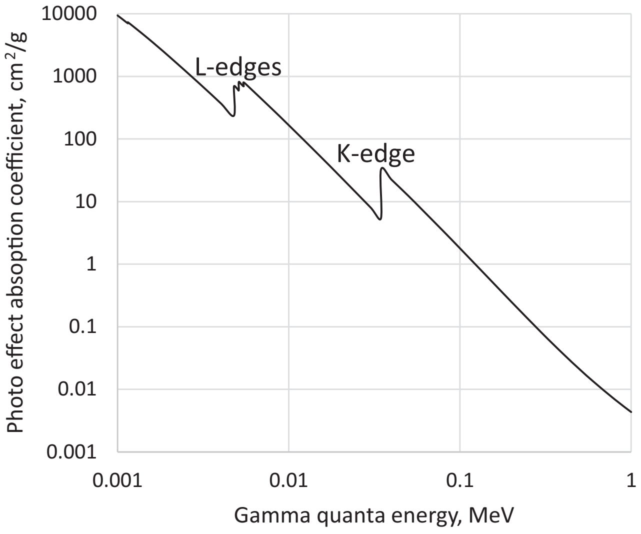

FIGURE 6.2 Schematic dependence of the photoeffect absorption coefficient in Xe on quantum energy.

K-shell becomes energetically impossible, and it is omitted from the process. A sharp vertical jump is observed on the curve, called the K-edge of absorption. At gamma quantum energy, which is smaller than the energy of the K-edge, only L-, M-, and the following shells participate in the photoelectric effect. As is known, these shells have subshells in accordance with different values of the orbital and total moments. The L-shell has three subshells, and the M-shell has five subshells; the absorption edge is divided into several ones.

As the energy of the gamma quantum decreases below the K-edge, the cross section again grows until an L_I-edge appears, followed by L_{II}- and L_{III}-edges, and so on. As the L-shell is switched off by parts, the magnitude of each jump is noticeably smaller than the jump corresponding to the K-edge of the absorption.

In Table 6.1, the energy values of the K-, L-, and M-edges of absorption of some elements are shown.

The angular distribution of photoelectrons is shown in Fig. 6.3. At low energies ($h\nu << mc^2$), photoelectrons are emitted predominantly in the direction of the electric vector of the incident electromagnetic wave, i.e., at the right angles to the direction of propagation of radiation. With increasing energy, the angular distribution extends forward.

As a result of photoeffect, a photoelectron appears with the energy

$$E = h\nu - U_{K,L,M\ldots}, \tag{6.3}$$

where $U_{K,\ L,\ M\ldots}$ is the binding energy of electrons on K-, L-, or M-shells.

After a photoelectron emerges out of the corresponding shell of an atom, with the greatest probability out of K-shell, a vacancy forms, followed by the cascade of transitions described in Section 4.6.

The cross sections of the photoelectric effect in a wide energy range for the air and lead are shown at the end of this chapter in Fig. 6.8.

TABLE 6.1 Values of the Energy of the K-, L-, and M-Edges of Absorption of Certain Elements (keV)

	Al	Si	Fe	Cu	Ge	Ag	I	Xe	Cs	Pb
K	1.56	1.84	7.11	8.98	11.1	25.5	33.5	34.7	36.0	88.0
L_I				1.1	1.41	3.8	5.2	5.5	5.7	15.9
L_{II}					1.25	3.5	4.9	5.1	5.4	15.2
L_{III}					1.22	3.4	4.6	4.8	5.0	13.0
M_I									1.2	3.9

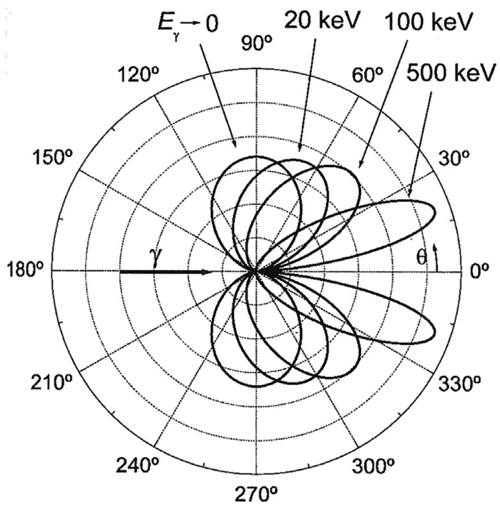

FIGURE 6.3 Angular distribution of photoelectrons.

6.3 GAMMA QUANTA SCATTERING BY FREE ELECTRONS (THE THOMSON SCATTERING)

Immediately after the discovery of X-rays, numerous attempts were made to obtain X-ray reflection from the mirror. These attempts were unsuccessful, but it was found that radiation is scattered almost in all directions. J. J. Thomson explained this process as a scattering by free electrons and obtained an expression for the differential cross section.

As is well known from electrodynamics, the intensity of dipole radiation in an element of solid angle per time unit is given by expression

$$dE_{rad} = \left(d^2P/dt^2\right)\sin^2 \varphi d\Omega/4\pi c^3, \tag{6.4}$$

where $P = er$ is the dipole moment of the system of charges, φ is the angle between the direction of scattering and the direction of the electric field vector of the incident wave.

The total intensity of the dipole radiation is

$$E_{rad} = 2e^2a^2/3c^3 \tag{6.5}$$

As the equation of motion of a charge has a view

$$m(dr/dt)^2 = eE, \tag{6.6}$$

then for the dipole moment one can obtain

$$d^2P/dt^2 = e^2E/m. \tag{6.7}$$

Substituting Eq. (6.7) into Eq. (6.4) one can find

$$dE_{rad} = e^4E^2 \sin^2\varphi d\Omega/4\pi m^2c^3 \tag{6.8}$$

The intensity of the incident radiation (the Pointing vector) is

$$E_{inc} = cE^2/4\pi \tag{6.9}$$

According to the definition of cross sections,

$$d_e\sigma_T = dE_{rad}/E_{inc} = \left(e^2/mc^2\right)^2 \sin^2 \varphi d\Omega = r_e^2 \sin^2 \varphi d\Omega \tag{6.10}$$

where $r_e = e^2/mc^2 4\pi\varepsilon_0 = 2{,}8 \times 10^{-13}$ cm is the classical electron radius (Section 3.1.9.2).

Eq. (6.10) is obtained for linearly polarized radiation. Thomson's formula for unpolarized radiation is obtained by averaging Eq. (6.10) over all possible directions of the electric field vector of the wave in a plane perpendicular to the propagation direction of the incident wave

$$\left(\sin^2 \varphi\right)_{av} = \left(1 + \cos^2 \theta\right)/2 \tag{6.11}$$

Then,

$$d_e\sigma_T = \left(r_e^2/2\right)\left(1+\cos^2\theta\right)d\Omega. \tag{6.12}$$

This is the differential Thomson scattering cross section.

It is seen from Eq. (6.12) that twice more quanta are scattered forward or backward at angles 0 and 180 degrees, than at 90 degrees. The angular distribution of the scattered radiation is shown in Fig. 6.5 (curve $\alpha = 0$).

The total cross section is obtained by integrating Eq. (6.12) with respect to the angle θ from 0 to π

$${}_e\sigma_T = (8/3)\pi r_e^2 = 0.66\times10^{-24}cm^2. \tag{6.13}$$

The cross section does not depend on the energy of the incident quanta.

The Thomson scattering is the limiting case of incoherent Compton scattering (Section 6.4) for a low photon energy ($h\nu << mc^2$).

Normally, there are no free electrons in the matter. Therefore, the Thomson scattering is a model process and is of interest as a certain reference phenomenon for comparison with the Compton scattering.

6.4 THE COMPTON EFFECT

6.4.1 Introduction

When the energy of quanta exceeds the binding energy of electrons in atoms, incoherent electron scattering, known as the Compton effect, begins to play an important role. This scattering was discovered in the earlier studies, but it was explained by the American scientist Arthur Compton in 1923 after a series of key experiments. This effect is one of the bright manifestations of the particle properties of gamma quanta. For his discovery and interpretation of incoherent electron scattering, A. Compton was awarded the Nobel Prize in 1927.

The Compton scattering occurs by the outer electrons of the atoms. With a simplified analysis, the binding of these electrons can be neglected, as their binding energy is, as a rule, much lower than the energy of gamma quanta, and the electrons can be considered to be free. One obtains the basic formulas with this simplifying assumption, and then it is seen how the binding of electrons affects the basic laws of scattering.

6.4.2 Laws of Conservation of Energy and Momentum

The basic relations for the energies and scattering angles can be obtained from the laws of conservation of energy and momentum. Let us assume that the electrons are at rest and the quanta are particles with energy $h\nu$ and momentum $h\nu/c$.

The vector scattering diagram is shown in Fig. 6.4. The energy of the primary quantum is $h\nu$, the energy of the scattered quantum is $h\nu'$, and E_C and p_C are the energy and momentum of the Compton electron. θ is the scattering angle of a gamma quantum, and φ is the angle of emission of the Compton electron (recoil electron).

$$h\nu = h\nu' + E_C, \tag{6.14}$$

$$h\nu/c = (h\nu'/c)\cos\theta + p_c\cos\varphi, \tag{6.15}$$

$$0 = (h\nu'/c)\sin\theta - p_c\cos\varphi \tag{6.16}$$

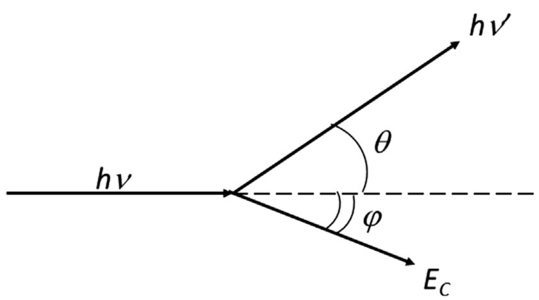

FIGURE 6.4 Vector diagram of the scattering of a gamma quantum on an electron.

Eq. (6.14) is the energy conservation law, and Eqs. (6.15) and (6.16) are the momentum conservation laws. Here the law of conservation of momentum is written in scalar form in two equations. Eq. (6.15) contains the projections of the momenta on the axis coinciding with the direction of motion of the primary quantum, and Eq. (6.16) is the projection onto the axis perpendicular to it.

Solving Eqs. (6.14)−(6.16) jointly, using the expression that connects the momentum and kinetic energy in the relativistic case, Eq. (3.150), and introducing the notation

$$h\nu/mc^2 = \alpha, \tag{6.17}$$

one can obtain the formulas for several parameters of the scattering process

$$h\nu' = \frac{h\nu}{1+\alpha(1-\cos\theta)}, \tag{6.18}$$

$$E_C = \frac{h\nu 2\alpha}{1+2\alpha+(1+\alpha^2)tg^2\varphi}, \tag{6.19}$$

$$E_C = \frac{h\nu\alpha(1-\cos\theta)}{1+\alpha(1-\cos\theta)}, \tag{6.20}$$

$$ctg\varphi = (1+\alpha)tg(\theta/2), \tag{6.21}$$

$$\cos\theta = 1-\frac{2}{(1+\alpha)^2 tg^2\varphi+1}. \tag{6.22}$$

Let us trace possible changes in the values of the parameters. The minimum value of the scattering angle is $\theta = 0$. This corresponds to $\varphi = \pi/2$, $h\nu = h\nu'$, $E_C = 0$. Such a collision is called a tangent. As the angle θ increases, the angle φ decreases, the energy of the scattered quantum $h\nu'$ decreases and the energy of the Compton electron E_C increases.

In the limit of a frontal collision, the scattered quantum flies backward**,** $\boldsymbol{\theta = \pi}$, and the Compton electron, which in this case receives the maximum energy, flies forward, $\varphi = 0$. Substituting these values into Eqs. (6.19) and (6.18), we find the maximum energy of the Compton electron

$$E_{C.\max} = h\nu 2\alpha/(1+2\alpha) \tag{6.23}$$

and the minimum energy of the scattered quantum

$$h\nu'_{\min} = h\nu/(1+2\alpha). \tag{6.24}$$

Note that for $h\nu >> mc^2$, $h\nu'_{min} = mc^2/2$ and $h\nu'$ $(90°) = mc^2$. In both cases, as well as for the angles of the intermediate one between them, the energy of the scattered quantum is practically independent of the energy of the primary quantum.

6.4.3 Differential Cross Sections of the Compton Effect

Before discussing the cross sections of the Compton effect, we note that, as already indicated in Section 3.1, the cross section is a proportionality coefficient in the relation connecting the result of the interaction with the initial product. Let us repeat here the relation (3.1)

$$dn = -\sigma nNdx.$$

Depending on the values denoted by the letters n and dn, Eq. (3.1) shows cross sections for the number of scattered and absorbed quanta, for the number of emitted electrons and for the energy of all possible components of the Compton scattering.

To understand the terminology used, let us point out that it is usually believed that the energy of scattered quanta is the scattered energy, as gamma radiation has good penetrating power and can emerge with a noticeable probability from the scatterer. The energy of the Compton electrons is the energy absorbed, as electrons have a relatively small range and are appreciably absorbed in the substance of the scatterer (absorber).

Let us make one more remark. The Compton scattering cross sections are calculated per electron. The atomic cross section for Compton scattering is Z times larger than the electron one

$$d_a\sigma = Zd_e\sigma. \tag{6.25}$$

Let us introduce the notation (the "s" sign denotes scattering, and the "a" sign means absorption in the superscript after the letter):

σ_γ is the cross section for the number of incident quanta,
σ_γ^s is the cross section for the number of scattered quanta,
σ_e is the cross section for the number of Compton electrons,
σ_{en} is the cross section for the energy of the incident quanta,
σ_{en}^s is the cross section for the energy of scattered quanta, and
σ_{en}^a is the cross section for the Compton electron energy.

The number of scattered quanta and Compton electrons that arise is naturally equal. Therefore, the cross sections for the number of scattered quanta and for the number of electrons that arise are also equal. And these cross sections are equal to the cross section for scattering of energy carried away from the beam because with each scattered quantum from the primary beam, an energy equal to $h\nu$ is carried away. The energies of scattered quanta and Compton electrons are not equal, so the corresponding cross sections are different. Therefore, we can denote this cross section simply as σ without subscripts and to write

$$\sigma_\gamma = \sigma_\gamma^s = \sigma_e = \sigma_{en} = \sigma. \tag{6.26}$$

Besides

$$\sigma_{en}^s + \sigma_{en}^a = \sigma_{en}. \tag{6.27}$$

These relations apply to both the total and differential cross sections.

The differential cross section for Compton scattering of unpolarized gamma radiation on free electrons, the so-called Klein–Nishina–Tamm cross section, has the form

$$\frac{d\sigma}{d\Omega} = \left(\frac{r_e^2}{2}\right)\left(\frac{h\nu}{h\nu'}\right)\left[\left(\frac{h\nu}{h\nu'}\right) + \left(\frac{h\nu'}{h\nu}\right) - \sin^2\theta\right]. \tag{6.28}$$

Substituting the ratio $h\nu'/h\nu$ from Eq. (6.18) into Eq. (6.28), after simple transformations we obtain the differential cross section for the number of photons scattered to the unit solid angle $d\Omega$ as a function of the emission angle of the scattered quantum θ

$$\frac{d_e\sigma}{d\Omega} = \frac{r_e^2}{2[1+\alpha(1-\cos\theta)]}\left\{1+\cos^2\theta + \frac{\alpha^2(1-\cos\theta)^2}{1+\alpha(1-\cos\theta)}\right\}. \tag{6.29}$$

It is easy to see that for $\alpha \to 0$ ($h\nu \to 0$) the Compton cross section goes over into the Thomson cross section (Section 6.3).

The resulting distribution over the angles is shown in Fig. 6.5. It can be seen from the figure that already at an energy of 100 keV, and especially at higher energies, forward scattering dominates.

The elementary solid angles for gamma quanta and electrons are equal

$$d\Omega = 2\pi\sin\theta d\theta, \tag{6.30}$$

$$d\Omega' = 2\pi\sin\varphi d\varphi. \tag{6.31}$$

Therefore, the differential cross section for the number of electrons scattered at a unit solid angle $d\sigma/d\Omega'$, as a function of the emission angle of an electron, can be written in an obvious way

$$d\sigma/d\Omega' = (d\sigma/d\Omega)(d\Omega/d\Omega'). \tag{6.32}$$

The ratio of solid angles is obtained using Eqs. (6.30) and (6.31)

$$d\Omega/d\Omega' = \sin\theta d\theta/\sin\varphi d\varphi. \tag{6.33}$$

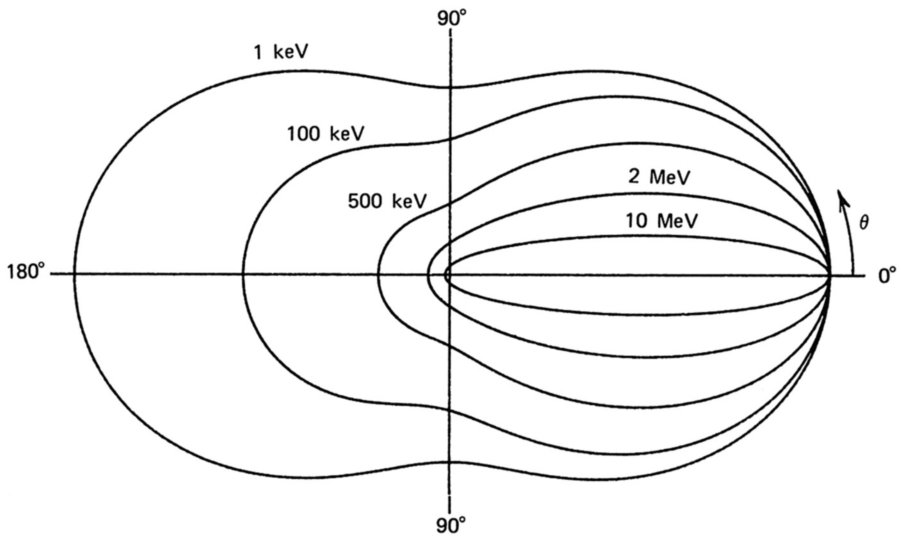

FIGURE 6.5 Angular distribution of scattered quanta. The figures are the energies of the incident quanta.

We differentiate Eq. (6.21), and after simple transformations using well-known trigonometric relations, we find

$$d\Omega/d\Omega' = -(1+\alpha)^{-1}(1+\cos\theta)(\sin\theta)\big/\sin^3\varphi \tag{6.34}$$

Expressing Eqs. (6.29) and (6.34) only as a function of the angle φ and substituting them in Eq. (6.32), we finally obtain

$$d\sigma/d\Omega' = \frac{2r_e^2(1+\alpha)^2}{(g+2\alpha)^2\cos^3\varphi}\left[1+\frac{(g-2)^2}{g^2}+\frac{4\alpha^2}{g(g+2\alpha)}\right], \tag{6.35}$$

where

$$g = (1+\alpha)^2 tg^2\varphi + 1. \tag{6.36}$$

The differential cross section for the energy interval of the scattered photon can be found using the obvious relation

$$\frac{d\sigma}{d(h\nu')} = \frac{d\sigma/d\Omega}{d\Omega/d(h\nu')} = \frac{d\sigma}{d\Omega}2\pi\sin\theta\frac{d\theta}{d(h\nu')}. \tag{6.37}$$

It is convenient to express $d\sigma/d(h\nu')$ as the dependence on $\alpha' = h\nu'/mc^2$. In such a way, the record is more compact

$$\frac{d\sigma}{d(\alpha')} = \frac{\pi r_e^2}{\alpha^2}\left[\frac{\alpha}{\alpha'}+\frac{\alpha'}{\alpha}-2\left(\frac{1}{\alpha'}-\frac{1}{\alpha}\right)+\left(\frac{1}{\alpha'}-\frac{1}{\alpha}\right)^2\right] \tag{6.38}$$

The differential cross section for the energy interval of the Compton electron is obtained in a similar manner

$$\frac{d\sigma}{dE_C} = \frac{d\sigma/d\Omega}{d\Omega/dE_k} = \frac{d\sigma}{d\Omega}2\pi\sin\theta\frac{d\theta}{dE_k}. \tag{6.39}$$

From here,

$$\frac{d\sigma}{dE_C} = \frac{\pi r_e^2}{\alpha^2 mc^2}\left\{2+\left[\frac{E_C}{h\nu-E_C}\right]^2\left(\alpha^{-2}+\left(\frac{h\nu-E_C}{h\nu}\right)-\frac{2(h\nu-E_C)}{\alpha E_C}\right)\right\} \tag{6.40}$$

Energy distribution of Compton electrons is shown in Fig. 6.6.

The other possible differential cross sections can be calculated in a similar way. Let us calculate the differential cross section for the scattering of energy as a function of the emission angle of the scattered quantum. We have already calculated the differential cross section for the collision $d\sigma/d\Omega$, Eq. (6.29). Between the collision cross section (6.29) and the sought energy scattering cross section there is a simple relation

$$\frac{d_e\sigma_{en}^s}{d\Omega} = \frac{h\nu}{h\nu'}\frac{d_e\sigma}{d\Omega}. \tag{6.41}$$

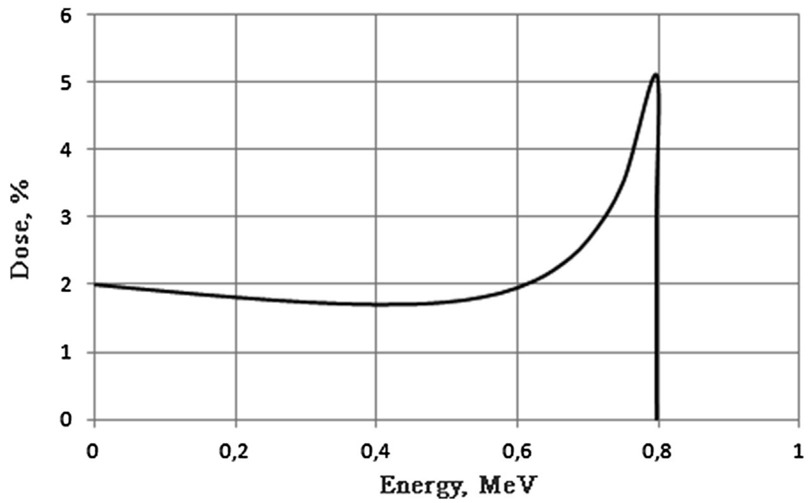

FIGURE 6.6 Energy distribution of Compton electrons for monoenergetic gamma quanta with energy 1 MeV. According to the formula (6.40).

Using Eq. (6.29) for $d\sigma/d\Omega$ and Eq. (6.18) for $h\nu/h\nu'$, we find

$$\frac{d\sigma_{en}^{s}}{d\Omega} = r_0^2(1+\alpha f)^{-3}\frac{(1+\cos^2\theta)}{2}\left[1+\frac{\alpha^2 f^2}{(1+\cos^2\theta)(1+\alpha f)}\right], \tag{6.42}$$

where $f = 1 - \cos\theta$.

The differential energy absorption cross section as a function of the emission angle of the Compton electron cannot be written out explicitly. It is easy to obtain it by using Eq. (6.35) for $d\sigma/d\Omega'$ and Eq. (6.18) for $h\nu/h\nu'$. Between the required energy absorption cross section and the collision cross section, Eq. (6.29), there is a simple relation analogous to Eq. (6.41)

$$\frac{d\sigma_{en}^{a}}{d\Omega'} = \frac{h\nu}{h\nu'}\frac{d\sigma}{d\Omega'}. \tag{6.43}$$

6.4.4 Total Cross Sections of the Compton Effect

Total cross sections are found by integrating the corresponding differential cross sections. Usually, the total cross section of the Compton collision and the total scattering and absorption cross sections are of interest.

The total collision cross section is found by integrating Eq. (6.29) over all possible values of the angle θ from 0 to π. As a result, we obtain

$${}_e\sigma = 2\pi r_e^2\left\{\left(a/\alpha^2\right)\left[(2a/b) - (1/\alpha)\ln b\right] + (1/2\alpha)\ln b - c/b^2\right\} \tag{6.44}$$

where $1+\alpha = a$, $1+2\alpha = b$, and $1+3\alpha = c$.

The total energy scattering cross section is obtained by integrating Eq. (6.42) over all possible values of the angle θ from 0 to π. As a result, we obtain

$$\sigma_{en}^{s} = \pi r_e^2\left\{\left(1/\alpha^3\right)\ln b + \left[2a\left(2\alpha^3 - 2\alpha - 1\right)/\alpha^2 b^2\right] + 8\alpha^2/b^2\right\}. \tag{6.45}$$

The total energy absorption cross section can be found by integrating Eq. (6.43) with respect to φ from 0 to $\pi/2$ or from

$$\sigma_{en}^{a} = \sigma - \sigma_{en}^{s}. \tag{6.46}$$

6.4.5 Scattering by the Bound Electrons

All the above expressions for the cross sections are obtained from the energy and momentum conservation laws written for the collision of a gamma quantum with a free electron and the original Klein–Nishina–Tamm cross section, Eq. (6.28), which is also derived for free electrons. However, in many types of substances, there are no free electrons, and all electrons are bound in atoms. Therefore, the above expressions are valid only when the binding energy is small in comparison with the energy obtained by the Compton electron as a result of the collision, which occurs only for valence electrons, the electrons of the outer shells. However, gamma quanta can be scattered on the electrons of all shells. For internal shells, at small scattering angles and for low-energy gamma quanta, the scattering probability decreases, so when summing the contribution of different electrons, this effect must be taken into account.

Summation essentially depends on how the electron perceives the energy. In those cases when the electron itself can take gamma quantum energy, each electron dissipates independently and the transition from the electron scattering cross section to the atomic scattering cross section is given by the formula

$$ {}_{a}\sigma_{noncog} = {}_{e}\sigma Zf \tag{6.47}$$

where f is the correction factor, called the incoherent scattering function.

The correction factor f for the calculated cross sections depends on the scattering angle, the quantum energy, and the atomic number of the substance.

For scattering of high-energy quanta by weakly bound electrons in substances with small Z and for scattering at large angles, the correction factor f tends to a maximum value equal to unity. In the opposite case, it tends to zero.

As the intensity of the incoherent scattered radiation decreases, the intensity of the coherent radiation, also called the Rayleigh radiation, increases (Section 6.5).

The recoil electrons, knocked out by monoenergetic gamma quanta, that fly out at a certain angle, in accordance with Eq. (6.20), must be monoenergetic, if they were at rest before the collision, as was assumed in the derivation of Eq. (6.20). However, some distribution of the electron energy is actually observed. The width of the distribution is in good agreement with the assumption that the gamma quanta collide not with resting electrons but with electrons moving in an orbit, and collisions with both the runaway and the oncoming electrons are possible. For the K-shell of Al, for $h\nu = 2mc^2$, the spread reaches 10%.

6.5 SCATTERING BY ATOMS (RAYLEIGH COHERENT SCATTERING)

For strongly bound electrons, and in conditions when only small energy could be transferred to them, the electrons cannot accept this energy, and the atom receives it entirely. As a result, the energy of the scattered quantum does not change. In this case, the recoil energy is perceived by the atom as a whole, and as the mass of the atom is very large, then, in accordance with the law of conservation of momentum, the quantum practically does not transmit its energy to the atom. Therefore, the scattered quantum has the same energy as the incident quantum.

The radiation scattered by different electrons of one atom interferes, and then the amplitudes are summed up, and only then the total intensity of the atom scattering is calculated, which, as is known, is proportional to the square of the amplitude. Such radiation is called coherent one. In the transition from the scattering cross section by the electron to the scattering by the whole atom, it is necessary to sum up the scattering intensities, and not the amplitudes. Such scattering is called incoherent.

In Section 6.4, the so-called Compton scattering, in which the energy of the scattered quanta differs from the primary energy, was considered in detail. However, experimentally in the spectrum of scattered radiation, in addition to quanta with an energy lower than the primary one, quanta with an energy equal to the energy of the incident quanta are observed. Such scattering has been just called the Rayleigh scattering.

With the above considerations, one can expect that in heavy atoms, where there are more strongly bound electrons and only external electrons are weakly bound, the relative role of the Rayleigh scattering will be greater. With decreasing atomic number, the role of the Rayleigh scattering decreases in comparison with the Compton scattering.

Theoretical calculations show that the Rayleigh scattering is important only for small angles and is negligible for angles larger than 60 degrees. About 70% of the coherently scattered radiation is concentrated within the scattering angles from $\theta = 0$ to a certain characteristic angle θ_0

$$\theta_0 = 2\arcsin\left[0.02Z^{1/3}\alpha^{-1}\right] \tag{6.48}$$

In lead, at a quantum energy $h\nu = 0.4$ MeV, $\theta_0 = 16°$, and for $h\nu = 1$ MeV, $\theta_0 = 6°$. The scattering cross section for small angles, $\theta < \theta_0$, is proportional to $(Z/h\nu)^2$ and weakly depends on the energy. For $\theta > \theta_0$, $d_a\sigma \sim (Z/h\nu)^3$. The Rayleigh scattering cross section is shown in Fig. 6.8. For heavy elements in the energy region less than 100 keV, it exceeds the Compton scattering cross section. For light elements, the region dominated by the Rayleigh scattering is shifted to the region of lower energies. In this energy range, it is the Rayleigh scattering that determines the nature and existence of the scattered radiation. As no energy transfer occurs from the photon to electrons, the Rayleigh scattering plays no role in the energy transfer coefficient; however, it contributes to the attenuation coefficient.

6.6 ELECTRON–POSITRON PAIR PRODUCTION

At a sufficiently high energy exceeding a certain threshold ($E_\gamma > 2m_ec^2$), a gamma quantum in matter can produce an electron–positron pair. In a vacuum, this process is forbidden because in this case it is impossible to satisfy the law of the momentum conservation. In matter, the pair production takes place in the field of the nucleus or in the field of the electron, which assumes the recoil momentum.

If the formation of a pair occurs in the nuclear field, then the recoil of the nucleus is extremely insignificant, and for the kinetic energy of the emerging particles (E_- is electron energy, E_+ is positron energy), one can write

$$E_- + E_+ = h\nu - 2mc^2 \tag{6.49}$$

where $2m_ec^2$ is the energy expended on the formation of the masses of two particles—an electron and a positron. Thus, $2mc^2$ is the threshold energy of pair production on the nucleus.

The threshold energy of pair production in an electron field is $4mc^2$. At the same time, three particles emerge: electron and positron as the components of the pair and the recoil electron which receives noticeable energy. Such an event is called a triplet. One of the triplet electrons, the recoil electron, has, as a rule, less energy and range than the other two.

The components of the pair fly forward in the direction of motion of the primary quantum, scattering at an angle

$$\theta \approx mc^2/h\nu \tag{6.50}$$

The energy distribution between the components of the pair is presented in Fig. 6.7. The graph shows that the positron (and, hence, the electron) has an almost equal probability of obtaining any energy in the interval from zero to the maximum energy transferred to the kinetic energy of the particles of the pair.

The general formula describing the process of pair production is too complex, and there is no reason to give it here. Private cases have limited values. We only note that the total cross section for the production of pairs per atom depends on the atomic number of the substance as

$$_a\sigma \sim Z^2 \tag{6.51}$$

At energies slightly above the threshold, the cross section increases in proportion to the logarithm of the energy, and at high energies ($h\nu \geq 5$ GeV), as a result of saturation associated with the screening of the nucleus by atomic electrons, it reaches saturation.

The production of pairs on electrons begins at $h\nu \geq 4mc^2$ and on nuclei with $h\nu \geq 2mc^2$. Therefore, in the interval $2mc^2 \leq h\nu \leq 4mc^2$, the pairs are formed only on nuclei. With increasing energy, the production of pairs on electrons first plays an insignificant role, and at high energies, the pair production in the field of electrons occurs Z times less often than in the field of the nucleus. Thus, the total cross section for pair production is proportional to $Z(Z+1)$. The dependence of the total cross section for the formation of pairs on energy is shown in Fig. 6.8.

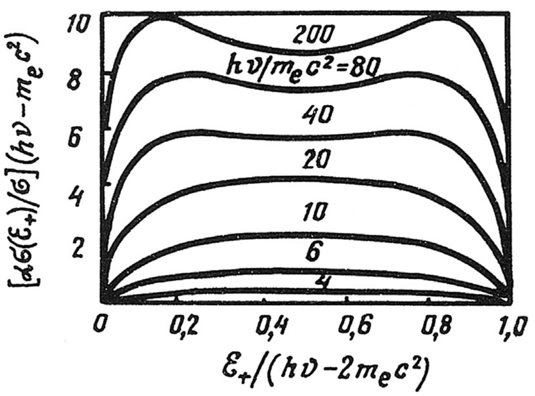

FIGURE 6.7 Energy distribution between the components of the pairs. The numbers on the curves are the energy of the incident gammas in the units of mc^2.

The production of pairs is essential in the development of electron−photon showers (Section 5.8).

Let us also consider the fate of the formed electrons and positrons. An electron loses energy in the usual way to ionization and, if it has a significant energy, to radiation. Having lost energy, an electron becomes one of the internal electrons of matter. Positron, although it has energy much greater than thermal, loses energy in much the same way as electron. However, at an energy close to the thermal one, i.e., kT, positron annihilates with one of the electrons of the substance. The annihilation process is rather complicated, and it passes, as a rule, through the stage of formation of a bound system of both electron and positron, the so-called positronium. For the question under consideration, it is important that positronium annihilates with an overwhelming probability in $\sim 10^{-6}$ s for two gamma quanta that fly apart in opposite directions. Each annihilation quantum has an energy equal to mc^2.

There are three options of the annihilation quanta fate.

First, both quanta can give out their energy within the same absorber in which the pair was born. The events are marked out, when the annihilation quanta are absorbed due to the photoelectric effect, i.e., completely. The time interval between the moment of pair creation and the absorption moments of annihilation quanta is determined by the lifetime of positronium and the passage of quanta. Obviously, this time is shorter than 1 μs. Thus, almost simultaneously in the absorber, an energy equal to the total energy of the primary gamma quantum is released. Partly, the energy spent for the formation of two particles returns on absorption of the annihilation quanta.

Second, a situation is possible in which one annihilation quantum flies out of the absorber, and the second one is absorbed. In this case, the energy in the absorber is equal to $(h\nu - mc^2)$.

Finally, both annihilation quanta can fly out of the absorber. Then, the energy in the absorber is equal to $h\nu - 2mc^2$.

Thus, in the process of pair formation three discrete energy release variants are possible: $h\nu$, $h\nu - mc^2$, and $h\nu - 2mc^2$. In addition, each annihilation quantum can undergo the Compton effect, and a scattered quantum can fly out of the absorber.

The probability of emission of annihilation quanta depends on the material and size of the absorber and on the geometry of the irradiation.

6.7 TOTAL ABSORPTION OF GAMMA RADIATION

The cross sections for the interaction of gamma radiation with matter can be specified in calculating not only per an electron ${}_e\sigma$ (cm^2/electron) or per an atom ${}_a\sigma$ (cm^2/atom) but also per 1 cm^3 of matter μ (cm^2/cm^3 = cm^{-1}) or 1 g of substance μ/ρ (cm^2/g).

The last two cross sections are called macroscopic cross sections or absorption coefficients. The relationship between these quantities is given by the formulas:

$$ {}_a\sigma = Z{}_e\sigma \tag{6.52} $$

$$ \mu = N_A \rho {}_a\sigma / A \tag{6.53} $$

$$ \mu/\rho = N_A {}_a\sigma / A \tag{6.54} $$

In the previous sections, the photoeffect and pair production cross sections are calculated per atom. In the photoeffect, the gamma quantum interacts only with internal electrons, mainly with two K-electrons. Therefore, if the cross section of the photoelectric effect on the K-shell is given for electron, then instead of formula (6.52), it is necessary to write ${}_a\sigma = 2{}_e\sigma$, regardless of the Z of the substance. The production of pairs proceeds mainly on nuclei, the number of which is equal to the number of atoms.

The Compton scattering cross sections are calculated per electron, because in this case the gamma quantum interacts with all the electrons of the atom.

As in the interaction of gamma quanta with matter, they either completely lose energy or are eliminated from the beam during scattering, and then the attenuation of the gamma radiation beam occurs exponentially

$$ n(x) = n_0 \exp(-\mu x) \tag{6.55} $$

where n is the flux density of gamma radiation passing through the absorber layer, n_0 is the flux density at the entrance to the absorber, and μ is the total absorption coefficient that accounts all three processes—photoeffect (μ_{ph}), the Compton effect (μ_C), and pair formation (μ_p)

$$ \mu = (N_A/A)\rho({}_a\sigma_{ph} + Z{}_e\sigma_C + {}_a\sigma_p) = \mu_{ph} + \mu_C + \mu_p \tag{6.56} $$

The coefficient μ in Eq. (6.56) characterizes the attenuation of the gamma radiation beam. If, however, it is necessary to take into account only the absorption of gamma radiation energy in the absorber, then in formula (6.56), instead of the collision cross section for the Compton effect ${}_e\sigma_C$, we must put the energy absorption cross section—$({}_e\sigma_{en}^a)_C$.

The number of gamma quanta that interacted in the absorber, per 1 cm^2, is equal to

$$n_{abs} = n_0[1 - \exp(-\mu x)] \tag{6.57}$$

The number of quanta absorbed in the absorber due to some specific process, e.g., due to the photoelectric effect, is equal to

$$n_{abs}^{ph} = n_0(\mu_{ph}/\mu)[1 - \exp(-\mu x)] \tag{6.58}$$

In Fig. 6.8, the cross sections for all processes in carbon and lead in the dependence of quantum energy are shown.

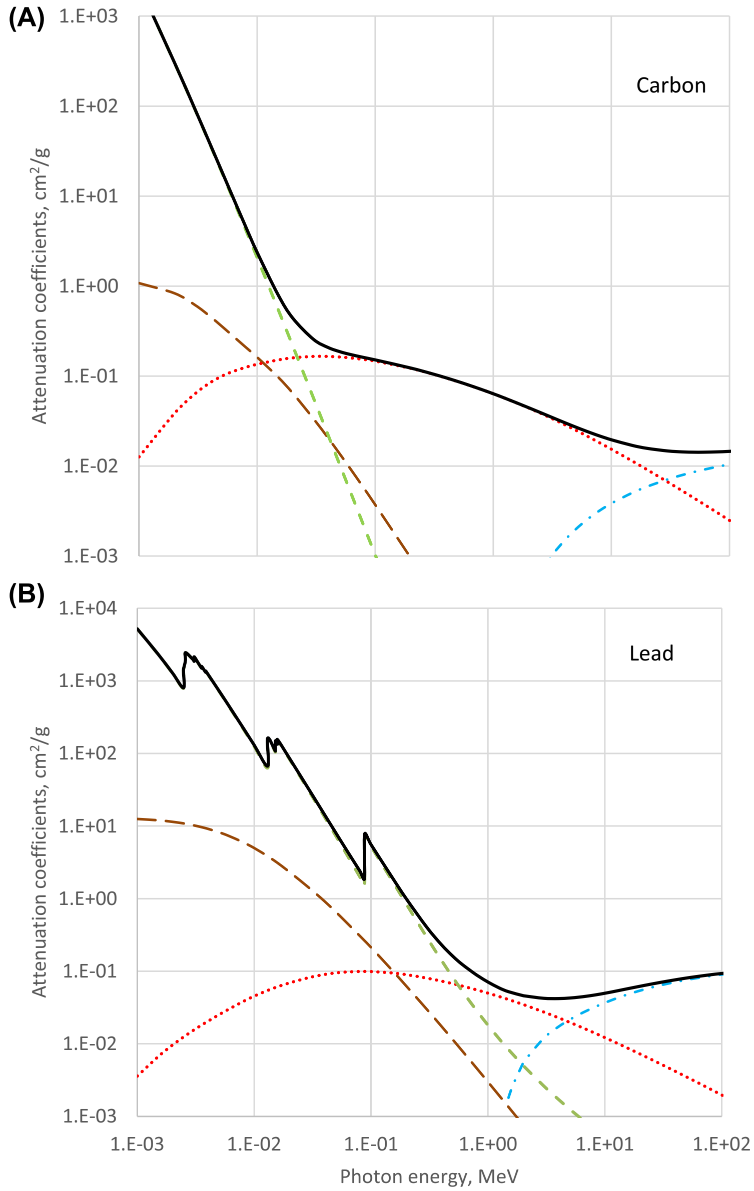

FIGURE 6.8 Attenuation coefficients of gamma radiation in carbon (A) and lead (B). Black solid line – total absorption, green dashed line – photo-effect, red dotted line – Compton scattering, green dotted-dashed line – pair production, red dashed line – Rayleigh scattering. Built by the author by the data of [National Institute of Standards and Technology. Physical Meas. Laboratory. XCOM. https://physics.nist.gov/PhysRefData/Xcom/html/xcom1.html]

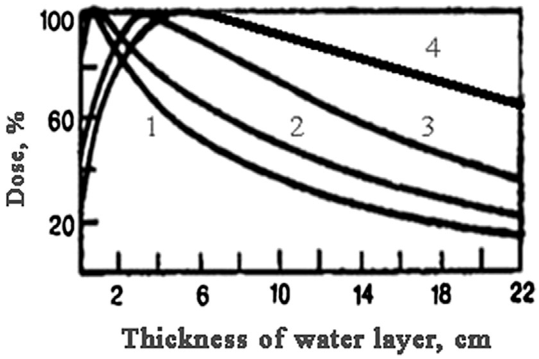

FIGURE 6.9 Distribution of the relative value of absorbed dose in dependence on the depth of penetration of gamma and X radiation in water. 1—X-rays with the energy 200 keV. 2—Gamma radiation ^{60}Co ($h\nu_{av}$ = 1.25 MeV). 3—Bremsstrahlung of electrons from betatron with the energy 15 MeV. 4—Bremsstrahlung of electrons from betatron with the energy 35 MeV. According to [43. Кудряшов Ю.Б., Беренфельд Б.С. Основы радиационной биофизики. Изд. МГУ, 1982, 305 с.].

6.8 BUILD-UP FACTOR

The above formulas, Eqs. (6.55)−(6.58), consider the passage through the substance of only the primary radiation beam. However, in a beam of gamma quanta, there are also scattered quanta, which can bring considerable energy to the measurement point. The results of calculations using these formulas correspond to the experiment in the geometry of a narrow beam. In the geometry of a wide beam, quanta scattered in the absorber can also reach the measurement point. In some cases, the total number of quanta can be a lot more than primary quanta.

This effect is taken into account by introducing the so-called build-up factor *B* into the attenuation law of the primary beam, which is expressed by Eq. (6.55). Factor *B* depends on the atomic number *Z*, the energy of gamma quanta, and the quantity μd. Factor *B* can take into account the accumulation of the number of gamma quanta, the total energy of gamma radiation, or the absorbed dose in a layer behind the absorber of certain thickness. Thus, the law of attenuation for a parallel flux of gamma quanta takes the form:

$$I(d) = I_0 B(Z, h\nu, \mu d)\exp(-\mu d) \tag{6.59}$$

For certain values of the parameters, the contribution of the scattered radiation to the formation of the irradiation dose exceeds the fraction of the primary radiation, attenuated by the shielding, by 1-2 orders of magnitude. Scattered radiation can lead to the fact that at certain radiation energies and shielding thicknesses, the dose behind the shielding is higher than the dose without shielding.

The value of *B* can be obtained by the Monte Carlo method or can be calculated using analytical approximations [1].

6.9 DISTRIBUTION OF ABSORBED ENERGY IN MATTER

The number of gamma quanta and the energy of the gamma ray beam as they pass through matter decrease exponentially. But the energy transferred from the gamma ray beam to matter first increases from some initial value to a maximum and only then decreases exponentially. The deeper the position of the maximum in the thickness of the substance, the greater is the energy of gamma quanta. This can be seen in Fig. 6.9.

The existence of such maximum is explained by the fact that the angular distributions of scattered quanta, of electrons knocked out in the Compton effect process, and of photoelectrons stretched forward with increasing energy. At a high energy of gamma quanta, the effect of pair formation is added, with the components of the pair, electrons and positrons, also flying predominantly forward. At small depths, the accumulation of secondary electrons begins, then it increases until an equilibrium is established between the formation of secondary particles and their absorption. The maximum of the energy release is at the depth of up to several centimeters. Thus, ^{60}Co gamma rays lose 60% of all energy when the first 5−6 cm of tissue passes, and 35 MeV photons give out their maximum energy at the depth of 6−8 cm.

REFERENCES

[1] S. Tanaka, Y. Harima, H. Hirayama, S. Ban, T. Nakamura, T. Kosako, Annotated bibliography and discussion of gamma ray buildup factors, Int. J. App. Radiat. Isot. 39 (1988) 241.

[2] National Institute of Standards and Technology. Physical Meas. Laboratory. XCOM. https://physics.nist.gov/PhysRefData/Xcom/html/xcom1.html.