Phase Sensitive Faraday Rotation

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Can light propagating through a medium be influenced by the application of an external magnetic field? You may have observed optical activity in chiral molecules in the PHY 200 lab. The present experiment extends these concepts to magnetically induced birefringence through the historically important Faraday Effect, which reveals the rich interplay between optics and magnetism.

KEYWORDS

Polarization · Birefringence · Faraday rotation · Verdet constant · Phase sensitive detection · Jones Calculus · Laser · Helmholtz coil · Resonance in an RLC series circuit.

PRE-REQUISITE EXPERIMENT: Basic measurements with the Lock-in amplifier.

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1 Objectives

In this experiment, we will,

1. shed some light on the underlying mechanism of magnetically induced birefringence,
2. demonstrate the advantages of phase sensitive detection (PSD),
3. understand the mathematical formalism for polarized light and its manipulation,
4. build or use a source of magnetic field and measure the field strength using a commercial magnetometer, and,

5. calculate the Verdet constant of terbium gallium garnet (TGG) and of a diamagnetic liquid.

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2 Introduction

Q 1. What is polarization of light? Write down the electric field for linear and circular polarization. Also, show that linearly polarized light can be written as a sum of left and right circular light [1]?

2.1 Birefringence

Michael Faraday observed the relationship between electromagnetism and light in 1845. Faraday's observation gave birth to the field of **magneto optics**: the interaction of optical radiation with magnetic media or the interaction of light with an optically inactive medium placed inside a magnetic field. The atom can be viewed as a positive charge surrounded by an electron shell. The electrons are bound to the nucleus. These binding forces can be modeled through springs as shown in Figure (1).

For an anisotropic substance, the binding forces on the electron are anisotropic implying that the spring constant will be different in different directions: an electron displaced from its equilibrium position along one direction will oscillate with a different frequency than another

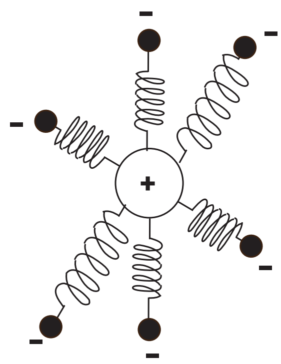

Figure 1: Negatively charged shell bound to positive nucleus by spring having different stiffness along different directions. This is also called the Lorentz model.

direction. Since the electric field associated with light drives the electrons of medium at its frequency, these electrons reradiate. The resulting secondary wavelets recombine and light propagates through a medium. The speed of the wave through the medium, is therefore, determined by the difference in natural resonating frequency of electrons and the frequency of the applied electric field. With anisotropy, the whole process becomes direction-dependent. Since the refractive index, $(n = c/v)$ is a function of speed, the anisotropy results in different refractive indices along different directions. This so-called *birefringence* manifests itself the in rotation of the plane of polarization [1].

2.2 Faraday rotation

Chiral compounds exhibit rotation of linearly polarized light due to natural birefringence, but this birefringence can also be induced in otherwise optically inactive materials either by applying stress, magnetic or electric field. Magnetically induced birefringence is called the Faraday effect.

Linearly polarized monochromatic light while transmitting through an optically inactive material, under the influence of an axial magnetic field, is rotated by an angle θ as shown in Figure 2. The angle of rotation θ is given by,

$$\theta = VBd, \tag{1}$$

provided the magnetic field remains uniform throughout the length d of sample. For non uniform magnetic field, θ is given by,

$$\theta = V \int_0^d B(z) dz. \tag{2}$$

The proportionality constant V is a characteristic of the material, called the **Verdet constant** and is a function of the wavelength of light, temperature and refractive index of the material. It is the rotation per unit path length per unit applied magnetic field. In other words, it quantifies the induced birefringence. In this experiment you will measure this induced birefringence.

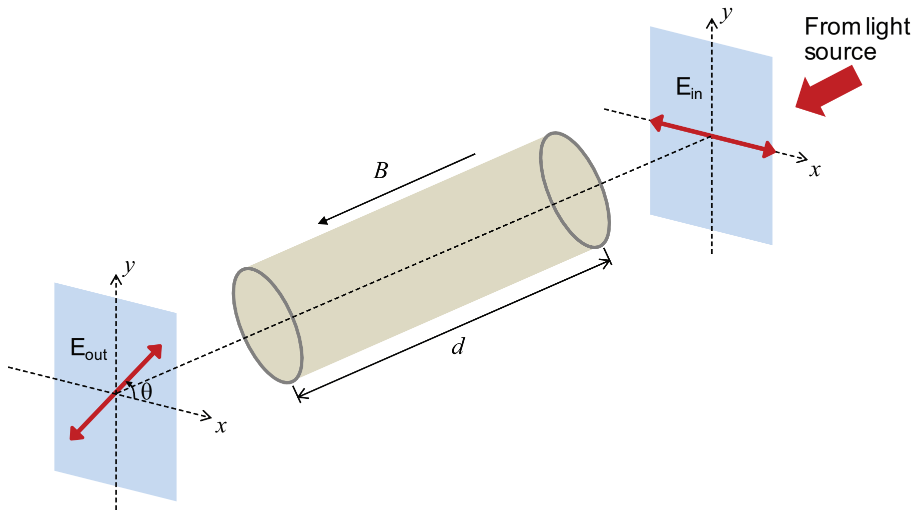

Figure 2: Faraday rotation, The plane of polarization of light is rotated under the action of an axial magnetic field.

2.3 Theoretical description

In the next few subsections, we will describe the origin of Faraday rotation for the avid reader.

2.3.1 Larmor precession of the electron cloud in an applied magnetic field

We now try to posit some foundational arguments describing the underlying mechanism of Faraday rotation. Consider an electron, moving in a circle of radius **r** in a plane whose normal makes an angle α with an applied magnetic field **B**. Since an electron is negatively charged its angular momentum **L** and magnetic moment $\boldsymbol{\mu_e}$ are opposite to each other. The magnetic field exerts a torque $\boldsymbol{\tau}$ on the magnetic dipole $\boldsymbol{\mu_e}$,

$$\boldsymbol{\tau} = \boldsymbol{\mu_e} \times \mathbf{B} = \mu_e B \sin\alpha.$$

Q 2. Referring to Figure 3 what is the direction of the torque on the magnetic dipole?

According to Newton's second law, an angular impulse $\boldsymbol{\tau}$ produces a change in angular momentum,

$$\boldsymbol{\tau} dt = d\mathbf{L}.$$

Thus, the attached vector **L** rotates in anticlockwise direction. The resulting precession traced out by tip of the vector **L** is shown in Figure 3. The angle of rotation through which angular momentum's projection along the applied field, $\mathbf{L}'$, moves in time dt is,

$$\begin{aligned} d\phi &= dL/L' \\ &= \tau dt/L \sin\alpha \end{aligned}$$

and the precessional or the Larmor angular velocity becomes,

$$\omega_L = \frac{d\phi}{dt} = \frac{\tau}{L\sin\alpha} = \frac{\mu_e B \sin\alpha}{L\sin\alpha} = \frac{\mu_e B}{L}. \quad (3)$$

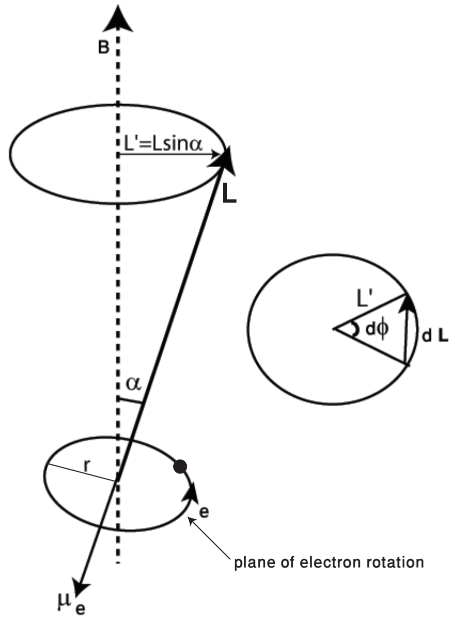

Figure 3: Precession of angular momentum vector about the direction of the applied magnetic field.

The magnetic moment of circular current is given by

$$\mu_e = iA = i(\pi r^2), \tag{4}$$

where,

$$i = \frac{e\omega}{2\pi}, \tag{5}$$

whereas the angular momentum of electron is given by,

$$\begin{aligned} \mathbf{L} &= \mathbf{r} \times \mathbf{p} \\ L &= mvr = mr^2\omega. \end{aligned} \tag{6}$$

Substituting Eqs (4), (5), (6) into (3), we obtain,

$$\omega_L = \left(\frac{e\omega}{2\pi}\right)\left(\frac{\pi r^2}{mr^2\omega}\right)B \tag{7}$$

$$= \frac{eB}{2m}, \tag{8}$$

showing that the Larmor frequency ω_L is independent of the orientation of the current loop and the overall effect is the Larmor precession of electronic structure about the direction of applied magnetic field [2].

2.3.2 Semi-Classical description of induced birefringence

You must have realized from Q 1 that plane polarized light is a combination of left and right circular (l and r) polarized light. Now, if light of vacuum frequency f is traveling through a medium whose electrons are rotating at the Larmor frequency then the l and r components will rotate the electron clouds with frequencies $f + f_L$ and $f - f_L$. Therefore in the dispersive medium, (refractive index is frequency dependent), the functional dependence of the respective refractive indices can be written as,

$$n_l = n(f - f_l)$$

and

$$n_r = n(f + f_l),$$

If plane polarized light traverses a distance d then the optical path lengths for l and r light are $n_l d$ and $n_r d$ respectively, so the optical path difference is $(n_r - n_l)d$. The difference of two refractive indices, the induced birefringence is,

$$n_r - n_l = n(f + f_L) - n(f - f_L)$$

From a Taylor series expansion,

$$\begin{aligned} n_r - n_l &\approx \left(n(f) + \frac{dn}{df} f_L\right) - \left(n(f) - \frac{dn}{df} f_L\right) && (9) \\ &= 2 f_L \frac{dn}{df}. && (10) \end{aligned}$$

From Eq 8,

$$f_L = \frac{\omega_L}{2\pi} = \frac{eB}{4\pi m},$$

and Eq (10) becomes,

$$n_r - n_l = 2\left(\frac{eB}{4\pi m}\right)\left(\frac{dn}{df}\right).$$

Since, phase change of a wave is k $(= 2\pi/\lambda)$ times the physical path traversed by the wave, the phase change for the two components is,

$$\begin{aligned} \phi_l &= \left(\frac{n_l d}{\lambda}\right)(2\pi) && (11) \\ \phi_r &= \left(\frac{n_r d}{\lambda}\right)(2\pi). && (12) \end{aligned}$$

When l and r waves enter the sample, the phase difference is zero, but the phase difference accumulates as light passes through the sample. The vector sum of the two electric fields on emerging from the sample is shown as **E** with a net rotation θ from its initial value. Since, **E** is an *equal* superposition of l and r components, we see from Figure (4) that,

$$\begin{aligned} \phi_l - \theta &= \phi_r + \theta \\ \Rightarrow \theta &= \frac{\phi_l - \phi_r}{2}. \end{aligned}$$

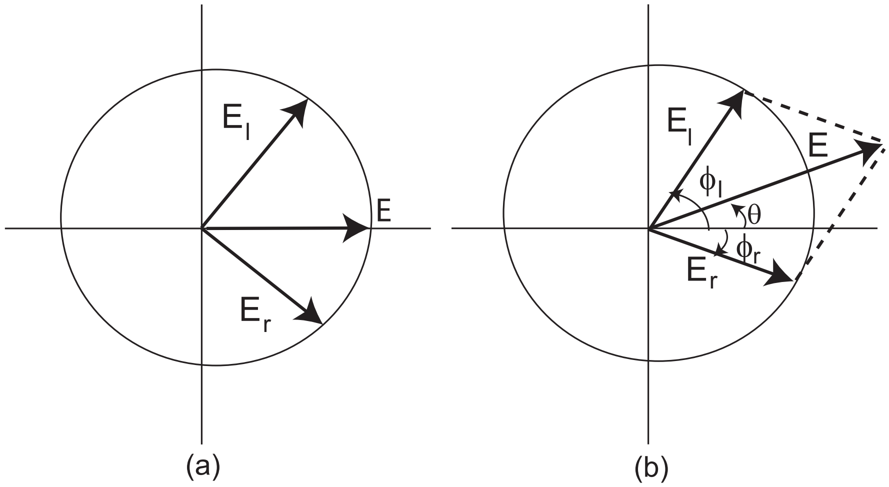

Figure 4: Linearly polarized light E can be decomposed into left and right linearly polarized components. Superposition of left and right circularly polarized light into linearly polarized light. (a) Before entering the sample, both the l and r components are moving with same speed albeit in opposite directions and (b) after emerging from the sample, these components have travelled with different velocities and acquired different phases.

Thus, the Faraday rotation angle is,

$$\begin{aligned}\theta &= \frac{1}{2}\left(\frac{2\pi d}{\lambda}\right)\left(n_l - nr\right) \\ &= \left(\frac{\pi d}{\lambda}\frac{eB}{2\pi m}\right)\left(\frac{dn}{df}\right) \\ &= \frac{e}{2m\lambda}\left(\frac{dn}{df}\right)Bd. \end{aligned} \tag{13}$$

Comparing Eq (1) and (13), the Verdet constant is,

$$V = \frac{e}{2m\lambda}\left(\frac{dn}{df}\right) \tag{14}$$

which is a function of wavelength and the dispersion [2]. The Faraday rotation is a direct result of $n_l \neq n_r$ arising because of the frequency dependent refractive index, dn/df.

2.3.3 Phenomenological description of Faraday effect based on Jones calculus

Jones calculus, invented by the American physicist R. Clark Jones, in 1941 [1], is a useful formalism to understand the state of polarization of perfectly polarized light as well as its transformation by various optical devices. For example, polarized light given by,

$$\mathbf{E}(z,t) = \hat{i}E_{ox}\cos(kz - \omega t + \phi_x) + \hat{j}E_{oy}\cos(kz - \omega t + \phi_y) \tag{15}$$

is represented in the Jones formalism as,

$$\tilde{\mathbf{E}}(z,t) = \begin{pmatrix} \tilde{\mathbf{E}}_x(z,t) \\ \tilde{\mathbf{E}}_y(z,t) \end{pmatrix} = \begin{pmatrix} E_{ox}e^{i\phi_x} \\ E_{oy}e^{i\phi_y} \end{pmatrix} e^{i(kz-\omega t)}. \tag{16}$$

The two component column vector completely specifies the amplitude and phase of electric field and hence its state of polarization. This is called the Jones vector. Most of the times, it is not necessary to know the exact phase but the phase difference $\varepsilon = \phi_x - \phi_x$ between the x and y components. Moreover, $e^{i(kz-\omega t)}$ is always understood to be present. Accordingly, Jones vector can also be written as,

$$\tilde{\mathbf{E}}(z,t) = \begin{pmatrix} E_{ox} \\ E_{oy}e^{i\varepsilon} \end{pmatrix} e^{i\phi_x}. \tag{17}$$

We choose not to write the term $e^{i\phi_x}$, since it does not have any physical consequences, remitting in the Jones vector,

$$\tilde{\mathbf{E}}(z,t) = \begin{pmatrix} E_{ox} \\ E_{oy}e^{i\varepsilon} \end{pmatrix}. \tag{18}$$

For linearly polarized light $\varepsilon = 0$ or 180^o, therefore the general form of Jones vector for linearly polarized light is,

$$\tilde{\mathbf{E}}(z,t) = \begin{pmatrix} E_{ox} \\ E_{oy} \end{pmatrix}. \tag{19}$$

Jones vectors can be normalized such that the sum of the squares of their components is 1, i.e,

$$E^*_{ox}E_{ox} + E^*_{oy}E_{oy} = 1.$$

Where the $*$ represents the complex conjugation. This normalized form discards the amplitude information needed for absorption calculations, but simplifies analysis in many other cases. The normalized form of (19) at an angle α w.r.t an arbitrary reference axis is,

$$\tilde{\mathbf{E}}(z,t) = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix},$$

where, the angle α is defined such that,

$$\begin{aligned} \cos\alpha &= \frac{E_{ox}}{\sqrt{E_{ox}^2 + E_{oy}^2}}, \\ &= E_{ox}, \\ \text{and,} \quad \sin\alpha &= \frac{E_{oy}}{\sqrt{E_{ox}^2 + E_{oy}^2}}, \\ &= E_{oy}. \end{aligned}$$

Q 3. Write down the normalized Jones column vector for horizontally, vertically, left and right circularly polarized light?

Suppose that the Jones vector for polarized incident beam $\tilde{\mathbf{E}}_i$ is represented by $\tilde{\mathbf{E}}_t$ after transmission through an optical element then, the optical element can be represented as a 2×2 transformation matrix J, called the Jones matrix. The transformation is captured through the relation,

$$\tilde{\mathbf{E}}_t = J\tilde{\mathbf{E}}_i \tag{20}$$

where

$$J = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}. \tag{21}$$

If the beam passes through a series of optical elements represented by the matrices $J_1, J_2, J_3, \ldots, J_n$, then

$$\tilde{\mathbf{E}}_t = J_n, \ldots, J_3, J_2, J_1 \tilde{\mathbf{E}}_i. \tag{22}$$

The matrices do not commute, so they must be applied in the proper order.

Q 4. Show that the transformation matrix J_h for a horizontal linear polarizer is

$$J_h = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{23}$$

Q 5. What is Malus's law? How does a polarizer work?

3 Experimental Technique

3.1 Why phase sensitive detection for measuring Faraday rotation?

We now turn to a description of how we will set out to measure Faraday rotation. You have already performed an introductory experiment of using the lock-in amplifier, so without discussing the details of the technique and the instrumentation any further, we will only focus on why are we using phase sensitive detection (PSD) in this experiment. Consider a simple optical system used to measure the transmission of light through a medium. Let us suppose

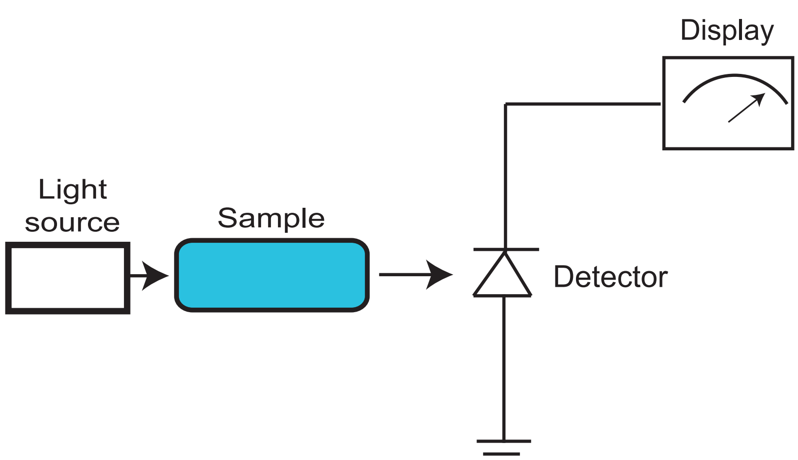

Figure 5: A simple optical system.

a small response obscured by overwhelming noise is to be measured. The output signal in this case will be,

$$V_o = V_{sig} + V_{noise}. \tag{24}$$

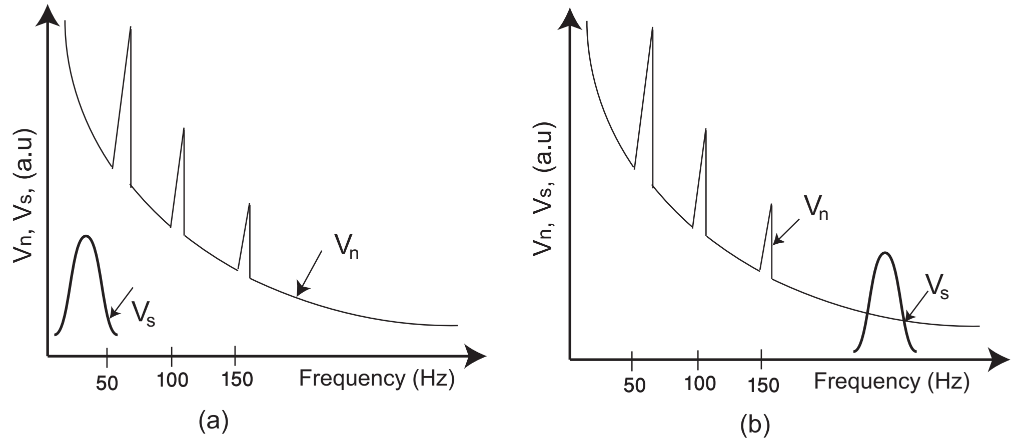

Figure 6: (a) Noise V_n and signal amplitude V_s as a function of frequency. (b) Modulating the signal to a region of low noise. The spikes in V_s represent 50 Hz and its harmonics.

The noise and signal amplitudes for such a system as a function of frequency are shown in Figure (6). The large peaks at 50 Hz and its multiples are due to electrical interference from the mains power lines. The noise power increases at lower frequencies (remember this is due to $1/f$ noise). Faraday rotation is extremely small in magnitude. If such a small signal that is overwhelmed by noise is to be measured, amplifying the signal will not improve the signal-to-noise ratio, since the noise is amplified with the signal. A clever approach is to move the signal to a region of low noise i.e., to a higher frequency. For example, in the present experiment, we use an ac magnetic field for inducing a time varying Faraday rotation. This means that our signal (the Faraday rotation) is shifted to a higher frequency part of the spectrum. This process is called modulation and is achieved by mixing the signal with a reference. (We hope you are already familiar with how a lock-in amplifier works). This technique gives two real advantages.

- The weak signal of interest buried in noise can be extracted successfully through PSD.
- Faraday rotation can be observed at smaller values of magnetic field (e.g., 80 G rms). This circumvents the need for large, expensive, bulky, water-cooled electromagnets for producing large DC magnetic fields.

3.2 The experiment

3.2.1 Malus's law

The plane of polarization of linearly polarized monochromatic light traversing through the sample S of length d placed under the influence of an ac magnetic field is rotated. Since the field is oscillatory, the rotation angle is also oscillatory. Another polarizer set at an arbitrary angle relative to input polarizer subsequent to the sample is required to analyze the rotation. The analyzer converts the polarization modulation to an amplitude modulation by the way of Malus's Law. The emerging light beam carrying the information in the form of amplitude variations is incident upon a photodiode whose output appears in the form of current proportional to the light intensity. In the first part of the experiment, we will seek to verify Malus's law. First, we present a treatment in the light of Jones calculus.

Let us suppose that the incident light polarized along the x-axis and is propagating in the z direction. The Jones vector for the electric field is,

$$\tilde{\mathbf{E}}_o = \begin{pmatrix} 1 \\ 0 \end{pmatrix} A_o \exp i(kz - wt) \tag{25}$$

where A_o corresponds to the amplitude of the electric field. Suppose, the analyzer is set at an angle ϕ w.r.t the polarizer. The Jones transformation matrix for the analyzer is,

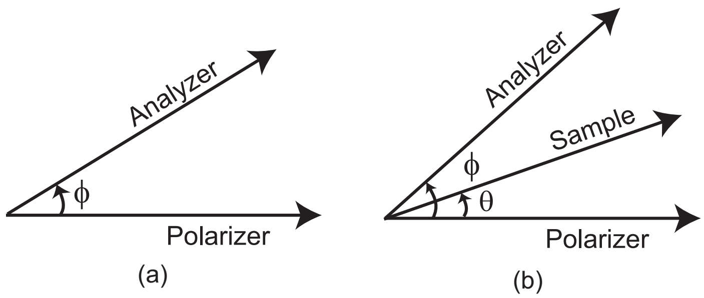

Figure 7: (a) The relative angle between the polarizer and the analyzer is ϕ. (b) Rotation θ by passing through the sample and detection after an analyzer placed at an angle ϕ.

$$J_{rot}(\phi) = \begin{pmatrix} \cos^2\phi & \sin\phi\cos\phi \\ \sin\phi\cos\phi & \sin^2\phi \end{pmatrix}. \tag{26}$$

Q 6. Show that $J_{rot}(\phi)$ in Eq (26) indeed represents the Jones matrix for an analyzer oriented at ϕ w.r.t the $x-$asis.
Q 7. Find the output intensity after an analyzer oriented at an angle ϕ.

3.2.2 Output intensity after a sample and analyzer arrangement

After passing through the sample S of length d placed in magnetic field, the plane of polarization of light is rotated by an angle θ, so the Jones vector after emerging from the sample is,

$$\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{27}$$

and the corresponding electric field is,

$$\tilde{\mathbf{E}} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} A_o \exp i(kz - \omega t). \tag{28}$$

Q 8. Since, the analyzer is set at an angle ϕ w.r.t the polarizer, show that electric field of the light beam after emerging from the sample followed by the analyzer is,

$$\tilde{\mathbf{E}} = \begin{pmatrix} \cos(\phi-\theta)\cos\phi \\ \cos(\phi-\theta)\sin\phi \end{pmatrix} A_o \exp i(kz - wt). \tag{29}$$

The intensity of light measured by the photodetector is,

$$I = MA_0^2[\cos^2(\phi - \theta)], \tag{30}$$

where M represents the relationship between the intensity and the sensitivity of the detection process. For all practical purposes, M can be considered to a scaling factor that is consistent over various experiments.

Q 9. Derive the expression (30).

Q 10. Write the Jones transformation matrix for the combination of the polarizer, sample and analyzer, placed in the same order.

3.2.3 Optimization of the analyzer angle

According to Eq. (30), the rotation of the plane of polarization manifests as a change in intensity at the photodiode. To get maximum change in intensity, the analyzer angle needs to be optimized. This means that we need to maximize $dI/d\phi$. Differentiating the intensity w.r.t ϕ, for a fixed Faraday rotation θ we obtain,

$$\frac{dI}{d\phi} = 2MA_0^2\cos(\phi-\theta)\sin(\phi-\theta) \tag{31}$$

$$= MA_0^2\sin(2(\phi-\theta)). \tag{32}$$

Differentiating again,

$$\frac{d^2I}{d^2\phi} = 2MA_0^2\cos\big(2(\phi-\theta)\big). \tag{33}$$

Maximum change in intensity is obtained by maximizing $dI/d\phi$ which is achieved by setting $d^2I/d\phi^2 = 0$, which implies that,

$$\phi - \theta = 45^o.$$

Since, the Faraday rotation θ is much smaller than ϕ, the maximum ΔI for any θ is obtained when the analyzer is set at 45^o relative to polarizer. The measured intensity, therefore, is,

$$\begin{aligned} I &= \frac{MA_0^2}{2}[1 + cos2(\phi-\theta)] \\ &= \frac{MA_0^2}{2}[1+\cos(2\phi)\cos(2\theta)+\sin(2\phi)\sin(2\theta)]. \end{aligned}$$

For $\phi = 45^o$ and a small angle θ, $\sin(2\theta) \cong (2\theta)$, leading to the observed intensity,

$$I \cong \frac{MA_0^2}{2}(1+2\theta). \tag{34}$$

3.2.4 Faraday rotation angle for oscillatory fields

The field is made oscillatory, with an oscillating frequency Ω,

$$B = B_0\sin(\Omega t),$$

then the angle $\theta(t)$ also becomes oscillatory. Since the angle of rotation is directly dependent on the magnetic field, i.e.,

$$\theta = \theta_0 \sin(\Omega t). \tag{35}$$

Therefore, for oscillatory fields Eq. (34) can be written as,

$$I \cong \frac{MA_0^2\big(1 + 2\theta_0 \sin(\Omega t)\big)}{2}. \tag{36}$$

We will use this modulating intensity for our phase sensitive detection scheme.

3.2.5 Converting light intensities into photocurrents

The photodiode converts the light intensities into current. We denoted intensities by I, so let's denote currents by i. In Eq. (36), the intensity has a DC component and a component that alternates at a frequency Ω, leading to,

$$i = i_{dc} + i_{ac}$$

where $i_{dc} = MA_0^2/2$ and $i_{ac} = M\theta_0 A_0^2 \sin(\Omega t)$. Modulated photocurrent due to Faraday rotation i_{ac} is measured through lock-in amplifier which displays the rms values. Therefore the output displayed on the front panel of the lock-in amplifier is,

$$i_{ac} = \frac{i_{ac}}{\sqrt{2}} = \frac{M\theta_0 A_0^2}{\sqrt{2}}.$$

Taking the ratio of the modulated current (shown by the lock-in amplifier) to the steady dc current, we obtain,

$$\frac{i_{ac}}{i_{dc}} = \frac{\theta_0 A_0^2}{\sqrt{2}}\frac{2}{A_0^2}, \tag{37}$$

and the Faraday rotation angle is determined by,

$$\theta_0 = \frac{i_{ac}}{\sqrt{2}\, i_{dc}} \tag{38}$$

The angle θ_0 corresponds to the peak field B_0 and $\theta_{\text{rms}} = \theta_0/\sqrt{2}$.

The dc component is measured by a voltmeter in the absence of magnetic field while the ac component is measured by the lock-in amplifier in the presence of the magnetic field. Finally assuming a uniform magnetic field, the Verdet constant is determined from the experimental values of θ, B and d,

$$\theta = VBd. \tag{39}$$

This equation could mean $\theta_0 = VB_0 d$ or $\theta_{\text{rms}} = VB_{\text{rms}} d$.

Q 11. What is the working principle of a photodetector? What does the photodetector measure, the electric field or the intensity?

At this stage, you must draw a conceptual sketch of how you will measure the Verdet constant. Your sketch should mention relative angles between the polarizer and the analyzer.

Table 1: List of major equipment used in the experiment.

Component	Specifications
Light source	laser 405 nm, 40 mW (B&W TEK (405-40E))
Magnetic field production	Signal generator, $10V_{pp}$ (BK Precision 4040DDS)
	Audio amplifier, 150 W
	Power supply, 10 A, 12 V
	Helmholtz coil, 120 G rms (homemade)
Detection element	Photodiode (New-port SLS-818)
Measuring instruments	Lock-in amplifier (Stanford Research System SR-830)
	Gaussmeter with axial and transverse probes (LakeShore 410)
	Clamp meter to measure AC current Kyoritsu
	Digital Multimeter
Others	Optical breadboard $90 \times 60 \times 6$ cm (homemade)
	Optical rail, 60 cm long (homemade)
	Rail carrier 2.5 cm (homemade)
	SS Post, 5 cm long (homemade)
	Post holder, 7.6 cm long (homemade)
	Rotation mount, 2 degree resolution (Thorlabs RSPO5/M)
	Laser post, 20 cm long
	Linear Polarizer extinction ratio=1000:1 (Thorlabs LPVIS050)
	V shaped laser housing (homemade)
	Crescent shaped cell holder (homemade)
	Teflon crystal holder (homemade)
	Laser safety glasses OD=4, OD=7 (Thorlabs LG4, LG10)
	M6 and M4 screws
	TGG crystal d=1 cm (Castech Inc.)

3.2.6 Overview of the experiment

Figure 8 shows the schematic diagram of the experimental setup for the observation of Faraday rotation.

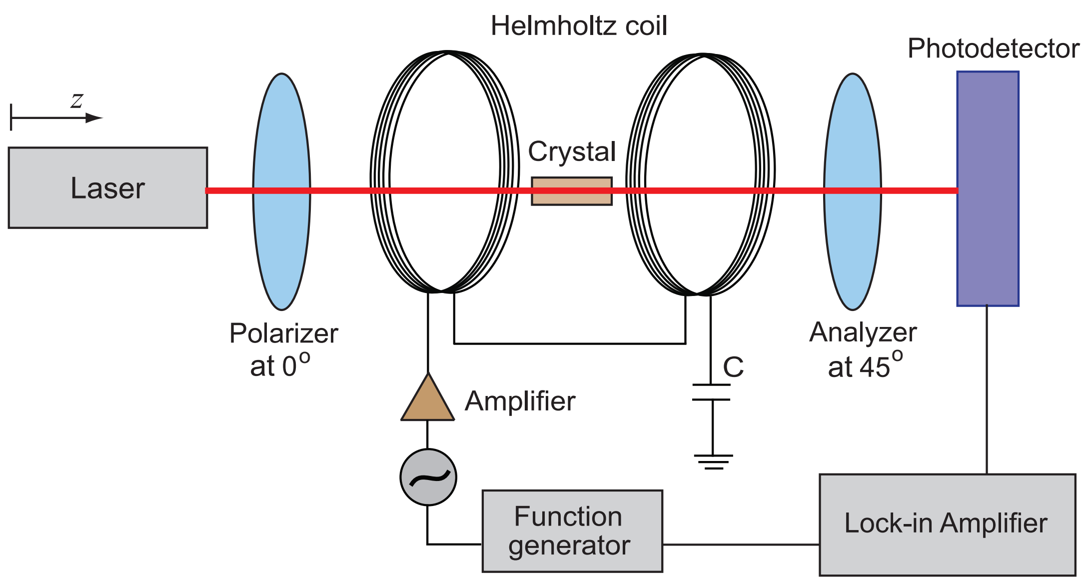

Figure 8: Schematic illustration of experimental setup for Faraday rotation. The dashed lines show the optical axes of the polarizer and the analyzer.

The setup comprises these modules:

(a) Light source: We use a diode-pumped solid laser of wavelength 405 nm,

(b) a mechanism for producing and measuring an oscillating magnetic field, and,

(c) detection and measurement equipment

Also see Table (1) which lists the most important components of this experiments.

3.2.7 Light source

The laser is of high power (~ 40 mW) and safety precautions must be taken. Safety goggles must be worn at all times, and there should be no exposure to stray light from the laser. The laser warms up in 15 minutes and should not be over run.

3.2.8 Mechanism for producing and detecting the magnetic field

In principle, both AC and DC magnetic field can be used in this experiment. DC sources include permanent magnets or solenoids possessing steady current in their windings. Since, Faraday rotation is extremely small in absolute magnitude, of the order of microradians, so a large DC magnetic field, of several kilo gauss will be required to achieve a sizeable rotation, which in turn requires large and bulky DC magnets or a large DC power supply to produce required field [5]. However, using an ac magnetic field, the rotation becomes oscillatory and can be tracked by PSD. For example, in this experiment, you will be provided with a Helmholtz coil capable of generating a field of ≈ 120 G rms at the centre.

In our experiment, the Helmholtz coil is constructed from 18 gauge copper wire (diameter 1.2 mm). Each multilayer coil consists of 18 turns in 18 layers, the coil's outer and inner diameters are 10.2 cm and 6.5 cm respectively. The length of each coil is 2.7 cm and radius is 4.5 cm. Inductance of the coils, determined using an LCR meter, is found to be 7 mH with a resistance of 1.5 Ω for each coil, so the total inductance of the Helmholtz pair is 15 mH and the total resistance is 3 Ω . The Helmholtz coil pair constitutes a series RLC circuit. At resonating frequency ω_r, the inductive reactance X_L is equal to the capacitive reactance X_L and total impedance is purely resistive. The resonance frequency is,

$$\omega_r = \sqrt{\frac{1}{LC}}$$

or

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}.$$

In our experiment, we will determine the resonant frequency, instead of relying on its predicted value.

Q 12. Calculate the resonating frequency when a capacitor of 1.2 μF is connected in series with the coil? Why is the Helmholtz coil made resonating?

3.2.9 Measurement of optical intensities

In section 3.2.4 the conversion of light intensities to photocurrents has already been discussed. This section will focus on the detector and the use of the lock-in amplifier in the measurement of these photocurrents. We have used the Newport (SLS-818) photodetector which will provide input to the lock-in. It is advisable to shield the detector from any stray magnetic fields in order to increase accuracy in our work and it is best to collimate the beam so that stray light does not affect our readings. The lock-in is the model SR-830 from Stanford Research systems. In order to ensure accuracy in work, care must be taken with the lock-in. The SR-830 adjusts itself automatically, however it is necessary to ensure that the correct parameters are already set. Sensitivity should be carefully selected, the shape of the reference wave must be set to the shape being provided by the function generator and the time constants should also be carefully set. The lock-in is the main tool which will measure the output $i_a c$.

3.2.10 Finding the resonant frequency of the Helmholtz coil

Assemble the setup according to Figure (8). Turn on the audio amplifier and the function generator. Amplify an approximately 1 V, 100 Hz sinusoidal signal through audio amplifier. Apply this amplified output to the Helmholtz coil. It is not necessary to turn on the laser for this step.

Increase the frequency of the ac signal applied to the coil. You may choose your step size (10-20 Hz). Tabulate the frequency against current passing through the Helmholtz coil and plot the frequency response. The current is measured with the help of a clamp meter. Determine the resonance frequency f_r.

3.2.11 Measuring the magnetic field produced by the Helmholtz coil

Set the function generator at the resonating frequency. Increase the current by increasing the amplitude of the signal from the function generator. Measure the magnetic field using the gaussmeter in ac mode (LakeShore, Model 410) equipped with transverse probe at the midpoint between the two coil. Plot a graph between current and magnetic field. Do you observe a linear relationship? Note that the gaussmeter measures the rms value of the magnetic field. At the end of this part, you must convince yourself that the current through the coil is a measure of the magnetic field. *Now turn off the magnetic field.*

3.2.12 Verifying Malus's Law

Next, you will verify the Malus's law. Turn on the laser and align the optics so that the laser spot falls right in the middle of the detector. Connect the I/V converter to the output of the photodetector. The photodetector produces a photocurrent proportional to the input light intensity. The I/V converter takes the photocurrent i and produces a voltage $|V| = iR_f$ where $R_f = 5.6\,\mathrm{M\Omega}$. Hence a measure of V, detected on a voltmeter, is reflection of the optical intensity.

Place a polarizer near the output of the laser and rotate its optical axis. Note down the intensity values. Now place a second polarizer infront of the photodetector. Vary the angle of the analyzer in steps of 20 and note down your results.

Plot a graph between the intensity and the analyzer angle. Fit the function using a suitable function. What are the outcomes of the fitting function?

3.2.13 Determining the Verdet constant of Terbium Gallium Garnet (TGG) crystal

You will now measure i_{dc}, the dc component of the detected signal, in the absence of magnetic field, but with the sample in place. Perform optical alignment again if required.

Fix the axial probe on the leftone holder on a side of the Helmholtz coil and turn on the Gauss meter. Select the 200 G range and ac mode. Switch on the audio amplifier and tune the function generator latter to the resonating frequency. Current is now passing through the Helmholtz coil. Select some value of current and measure the corresponding magnetic field at the midpoint between the Helmholtz coils (A field of 90 G rms is a reasonably good value). Now move the probe away from the center of coils on both sides. Check that magnetic field is not reaching the polarizers and photodiode. Switch off the magnetic field.

Now place the terbium gallium garnet (TGG) crystal on the crystal holder. The TGG crystal is 1 cm long. (**Note: Never touch the lateral surfaces of the TGG crystal.**)

Minimize the background reading from ambient light either by placing a black box around the detector to block ambient light. Open the laserhead shutter.

Rotate the analyzer angle ϕ and find out the maximum and minimum intensity. Set the analyzer at an angle of 45^o w.r.t the polarizer. Note down the value of voltage at the multimeter and divide it by R_f, the feedback resistance of the I/V converter, to obtain i_{dc}.

Next, you will determine i_{ac}, the (rms value of the) ac component of the photocurrent using the lock-in amplifier.

Activate the magnetic field. Turn on the function generator and select the resonant frequency. Connect the output of the function generator to the reference input of the lock-in. Select the current input mode *I*, from the input section. Connect the photodiode output directly (without the I/V converter) to the input BNC connector. Make sure that no error indication (unlock or overload) occurs. Suitable values for the lock-in are the sensitivity in the range of *n*A and a pre-time constant of 3 second.

Rotate the analyzer angle ϕ in steps of 10^o and tabulate i_{ac}, for any fixed value of the magnetic field. You will observe that the maximum rotation occurs when analyzer is at an angle of approximately 45^o relative to polarizer.

Q 13. What does the reading on the lock-in amplifier physically represent?

Now fix the analyzer at 45^o relative to the polarizer. Increase the magnetic field, from an initial value of 10 Gauss, in steps of 5 or 10 Gauss by increasing the current. For best results the field should not exceed 120 Gauss. The transverse probe of Gaussmeter can be fixed to observe the magnetic field near the center of Helmholtz coil. However, it will not give us the field *at* the position of the crystal. The field can be measured from the current through the Helmholtz coil.

Tabulate the values for i_{ac} for each value of magnetic field. Plot i_{ac} versus magnetic field.

Q 14. Plot the Faraday rotation and use the results to calculate the Verdet constant of your sample.

Q 15. Clearly quantify your uncertainties.

Q 16. Can you measure i_{dc} with the help of the lock-in amplifier?

4 Measurement of the the Verdet constant using higher harmonic components (OPTIONAL)

The light rotated by the Faraday medium incident on the photodetector after coming through the analyzer, contains fundamental as well as components at higher frequencies. The rms values u_1 and u_2 of the signal at frequencies f and $2f$, can be measured, where f is the frequency of ac signal passing through Helmholtz coil. The ratios u_1/U_o and u_2/U_o can also be used to determine the Verdet constant, where, U_o is the steady output from the photodiode under zero magnetic field and analyzer set for maximum transmittance i.e., analyzer is at 0°

[6]. The power transmitted through a Faraday rotator is,

$$
\begin{aligned}
I &= \frac{MA_o^2}{2}[1+\cos 2(\phi-\theta)] \qquad (40)\\
&= \frac{MA_o^2}{2}[1+\cos 2(\phi-\theta_o\cos(\Omega t))]\\
&= \frac{MA_o^2}{2}[1+\cos 2\phi\cos(2\theta_o\cos\Omega t)+\sin 2\phi\sin(2\theta_o\cos\Omega t)].
\end{aligned}
$$

Using the Jacobi-Anger expansion, we obtain[7],

$$
\begin{aligned}
\cos(2\theta_o\cos\Omega t) &= J_o(2\theta_o)+2\sum_{m=1}^{\infty}(-1)^m J_{2m}(2\theta_o)\cos(2m\Omega t)\\
\sin(2\theta_o\cos\Omega t) &= 2\sum_{m=1}^{\infty}(-1)^m J_{2m+1}(2\theta_o)\cos((2m+1)\Omega t)
\end{aligned}
$$

where the Bessel function is,

$$
J_\alpha(x) = \sum_{q=0}^{\infty}\frac{(-1)^q}{q!\gamma(q+\alpha+1)}(\frac{x}{2})^{2q+\alpha}
$$

and γ is the factorial function, given by,[1]

$$
\gamma(n) = (n-1)!
$$

Therefore, Eq (41) becomes,

$$
\begin{aligned}
I &= \frac{MA_o^2}{2}\Bigg[1+\cos(2\phi)\Bigg(J_o(2\theta_o)+2\sum_{m=1}^{\infty}(-1)^m J_{2m}(2\theta_o)\cos(2m\Omega t)\Bigg)\\
&\quad+\sin(2\phi)\Bigg(2\sum_{m=0}^{\infty}(-1)^m J_{2m+1}(2\theta_o)\cos(2m+1)\Omega t\Bigg)\Bigg]. \qquad (41)
\end{aligned}
$$

Let the amplitude of coefficient of the terms containing Ωt and $2\Omega t$ be represented by s_1 and s_2 respectively. Then,

$$
\begin{aligned}
s_1 &= \frac{MA_o^2}{2}2(-1)^0 J_1(2\theta_o)|\sin(2\phi)|\\
&= MA_o^2\sum_{q=0}^{\infty}\frac{(-1)^q}{q!\gamma(q+1+1)}\left[\frac{2\theta_o}{2}\right]^{2q+1}|\sin(2\phi)|\\
&= MA_o{}^2\left[\frac{(-1)^0}{0!\gamma(2)}\theta_o+\frac{-1}{1!\gamma(3)}\theta_o^3+\frac{(-1)^2}{2!\gamma(4)}\theta_o^5+\ldots\right]|\sin(2\phi)| \qquad (42)\\
&= MA_o{}^2\left[\frac{1}{1!}\theta_o+\frac{-1}{1!2!}\theta_o^3+\frac{1}{2!3!}\theta_o^5\ldots\right]|\sin(2\phi)|\\
&= MA_o{}^2\theta_o\left[1+\frac{-1}{2}\theta_o^2+\frac{1}{12}\theta_o^4\ldots\right]|\sin(2\phi)| \qquad (43)
\end{aligned}
$$

[1] Γ is the conventional symbol to generalize the factorial function. Since, we are using Γ for numerically integrated magnetic field, therefore, we have used γ to denote the general form of factorial function.

$$
\begin{aligned}
s_2 &= \frac{MA_o^2}{2} 2J_2(2\theta_o)|\cos(2\phi)| \\
&= MA_o^2 \sum_{q=0}^{\infty} \frac{(-1)^q}{q!\gamma(q+2+1)} \left[\frac{2\theta_o}{2}\right]^{2q+2} |\cos(2\phi)| \\
&= MA_o^2 \left[\frac{1}{\gamma(3)}\theta_o^2 + \frac{-1}{\gamma(4)}\theta_o^4 + \frac{(-1)^2}{2!\gamma(5)}\theta_o^6 + \ldots\right] |\cos(2\phi)| \\
&= A_o^2 \left[\frac{1}{2!}\theta_o^2 - \frac{1}{3!}\theta_o^4 + \frac{1}{2!4!}\theta_o^6 \ldots\right] |\cos(2\phi)| \\
&= \frac{MA_o^2}{2}\theta_o^2 \left[1 - \frac{1}{3}\theta_o^2 + \frac{1}{24}\theta_o^4 \ldots\right] |\cos(2\phi)| \qquad (44)
\end{aligned}
$$

Since,

$$
\theta_o = VB_o d \qquad (45)
$$

Substituting equation (45) in (43)

$$
\begin{aligned}
s_1 &= MA_o{}^2 VB_o d \left[1 + \frac{-1}{2}(VB_o d)^2 + \frac{1}{12}(VB_o d)^4 \ldots\right] |\sin(2\phi)| \\
&= U_o VB_o d \left[1 + \frac{-1}{2}(VB_o d)^2 + \frac{1}{12}(VB_o d)^4 \ldots\right] |\sin(2\phi)|. \qquad (46)
\end{aligned}
$$

where, U_o is the steady power on photodetector when polarizers are set for maximum transmittance (in the absence of applied magnetic field).

Substituting equation (45) into (44), we obtain,

$$
\begin{aligned}
s_2 &= \frac{MA_o^2}{2}(VB_o d)^2 \left[1 - \frac{1}{3}(VB_o d)^2 + \frac{1}{24}(VB_o d)^4 + \ldots\right] |\cos(2\phi)| \\
&= \frac{U_o}{2}(VB_o d)^2 \left[1 - \frac{1}{3}(VB_o d)^2 + \frac{1}{24}(VB_o d)^4 + \ldots\right] |\cos(2\phi)|. \qquad (47)
\end{aligned}
$$

The f and $2f$ components are determined through lock-in amplifier which displays rms values, so from equation (46), the rms value of the first harmonic component of output current (ignoring higher order terms) is,

$$
\begin{aligned}
u_1 &\approx \frac{U_o VB_o d}{\sqrt{2}} |\sin(2\phi)| \\
&= U_o VBd |\sin(2\phi)| \qquad (48)
\end{aligned}
$$

where $B = B_o/\sqrt{2}$, B represents the rms value of the field measured by the Gaussmeter. Similarly, from (47) the rms value of the second harmonic component of output current is,

$$
\begin{aligned}
u_2 &\approx \frac{U_o}{2\sqrt{2}}(VB_o d)^2 |\cos(2\phi)| \qquad (49) \\
&= \frac{U_o}{\sqrt{2}}(VBd)^2 |\cos(2\phi)|. \qquad (50)
\end{aligned}
$$

Both equations (48) and (50) can be used to determine Verdet constant.

In short, we have three three different means of measuring the Farday rotation,

Method 1. The gradient of the plot of u_1 or i_{ac} against $\mathbf{B}$ for $\phi = 45^o$ results in the Verdet constant. This is, in fact, the method you have used in previous section. Since, $U_o = 2i_{dc}$ and $u_1 = i_{ac}$, Equation (48) is actually Eq. (39) in disguise.

Method 2. Determine the gradient of the least squares-fit line to a plot of u_1/U_o against $|\sin 2\phi|$ for fixed B. Equate the gradient to VBd and find the Verdet constant [6].

Method 3. Determine the gradient of a plot of u_2 against $\mathbf{B}^2$ when $\phi = 90^o$. equate this to to $V^2 d^2 U_o/\sqrt{2}$ and find the Verdet constant.

Q 17. Find the Verdet constant for TGG at 405 nm using methods 2 and 3.