Hydrogen and Deuterium Spectroscopy

Abstract: An optical spectrometer equipped with calibrated transmission and reflection diffraction gratings with 602.9(2) and 620.1(1) lines/mm, respectively, has been used to determine a value for the Rydberg constant. Using the transmission grating, the angular positions of 7 Balmer emission lines from hydrogen were measured and from a least-squares fit to the data, *R* was determined to be 1.09755(8) 10^7m ^{1}. Using the reflection grating, 26 Balmer lines up to 5th order could be observed. From a least-squares fit to their positions, *R* was determined to be 1.09819(14) 10^7m ^{1}. Using the same reflection grating and a discharge tube containing both hydrogen and deuterium, the ratio of the mass of the proton to the electron was determined from the angular splitting of the H and D lines. The relative mass was determined to be 1200(400).

Transmission Grating Calibration: To calibrate the transmission grating, the angular positions of the four prominent emission lines from a Hg discharge tube were measured in first, second and third order both left and right of the straight-through position. The angular positions were as follows:

	order(m)	$_{Left}$(0.01°)	$_{Right}$(0.01°)	$_{Mean}$(0.01°)
violet	1	15.23°	15.21°	15.22°
green	1	19.23°	19.20°	19.21°
yellow 1	1	20.36°	20.35°	20.35°
yellow2	1	20.43°	20.41°	20.42°
violet	2	31.68°	31.68°	31.68°
green	2	41.15°	41.17°	41.16°
yellow 1	2	44.04°	44.07°	44.05°
yellow2	2	44.25°	44.27°	44.26°
violet	3	not obs	.04°	°

Using the literature values of the emission line wavelengths, a plot of sin against *m* gave a straight line (Figure 1), the gradient of which was equal to the line density of the grating. A least-squares fit the data using LINEST gave the line density as 602.9(2)lines/mm.

Rydberg Constant: Using the calibrated transmission grating, the angular positions of the violet, green and red Balmer lines from a hydrogen emission tube were measured in first and second order. The wavelengths of these lines were then determined using the diffraction equation for normal incidence

$$m \qquad d \sin$$

The measured wavelengths were		1st order	2nd order
	violet	433.8(3)nm	433.9(2)nm
	green	485.9(3)nm	486.0(2)nm
	red	655.8(3)nm	656.1(2)nm

The wavelengths of these lines are predicted by the Rydberg equation

$$\frac{1}{\ } \quad R \; \frac{1}{2^2} \quad \frac{1}{n^2}$$

A plot of 1/ versus (1/4-1/n^2) is shown in Figure 2. A least-squares fit to the data, constraining the fitted line to pass through the origin, gave a value for *R* of 1.09755(8) 10^7m ^{1}, in excellent agreement with the literature value of 1.09737 10^7m ^{1}.

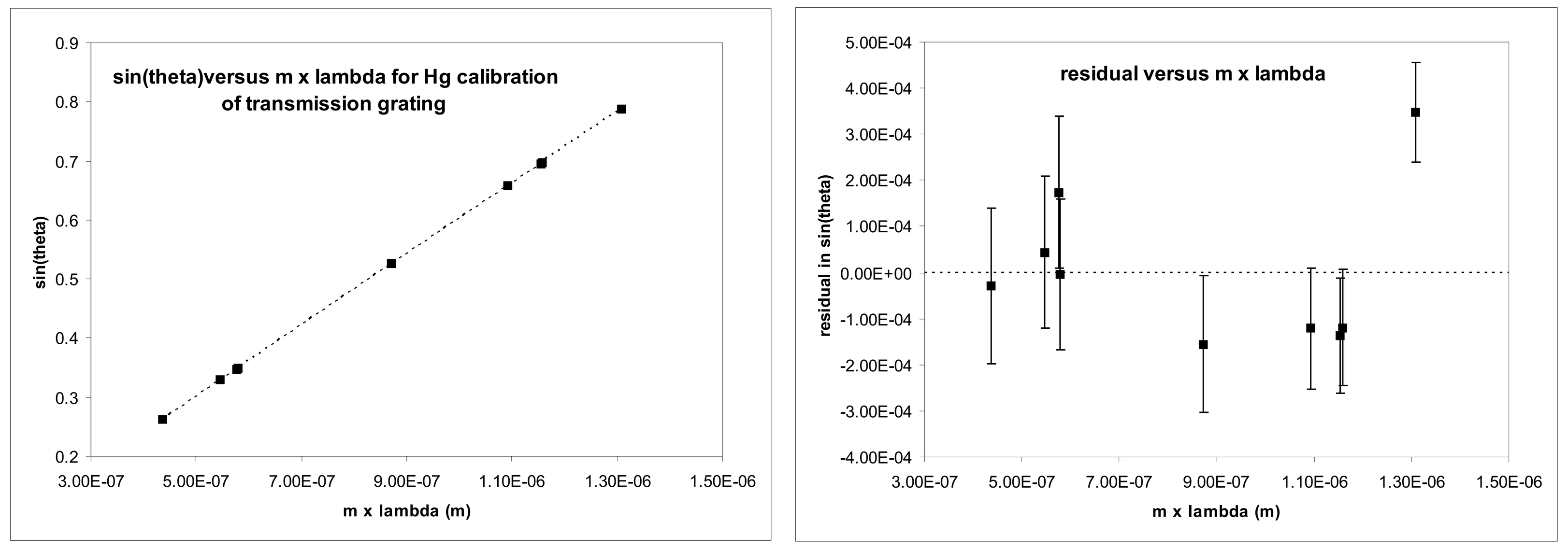

Figure 1: Plot and LS fit to the calibration data for the transmission grating. The error bars are smaller than the symbols used to plot the data. The right-hand graph shows a plot of the residuals from the fit.

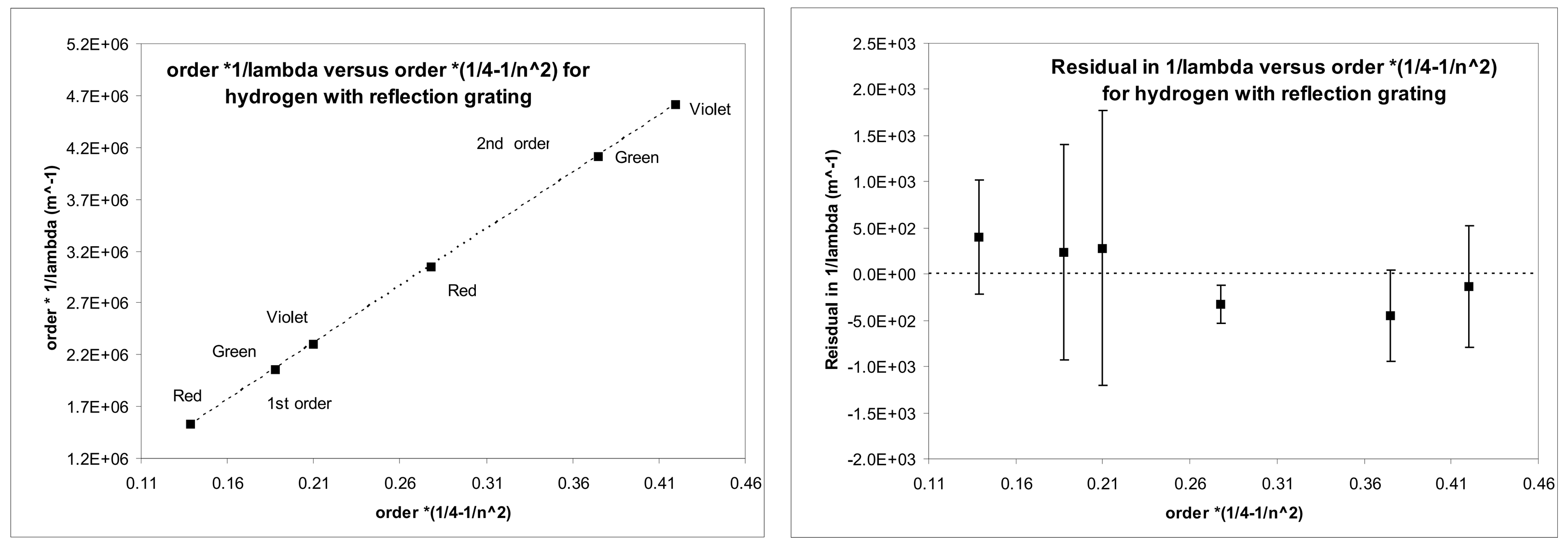

Figure 2: Plot of the measured wavelengths of the H emission lines versus (1/4–1/n^2). To separate the different orders, both *x* and *y* coordinates have been multiplied by the order of the emission line. The error bars are smaller than the symbols used to plot the data. The right-hand graph shows a plot of the residuals from the fit.

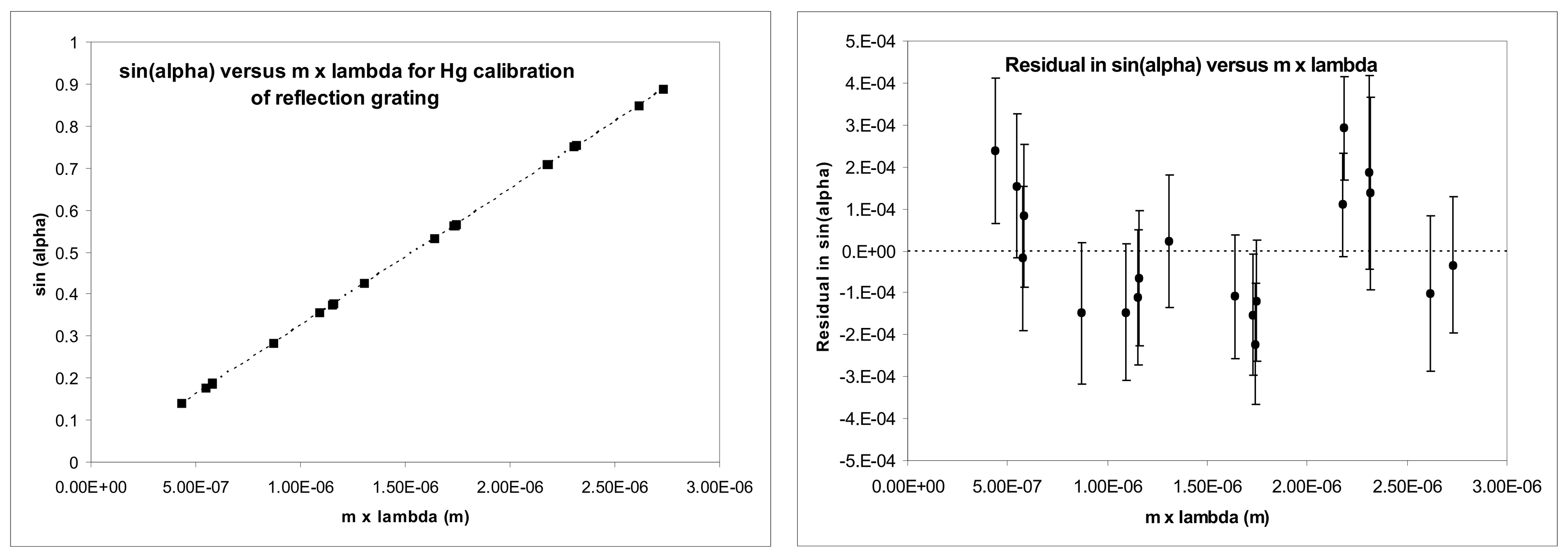

Figure 3: Plot and LS fit to the calibration data for the reflection grating. The error bars are smaller than the symbols used to plot the data. The right-hand graph shows a plot of the residuals from the fit.

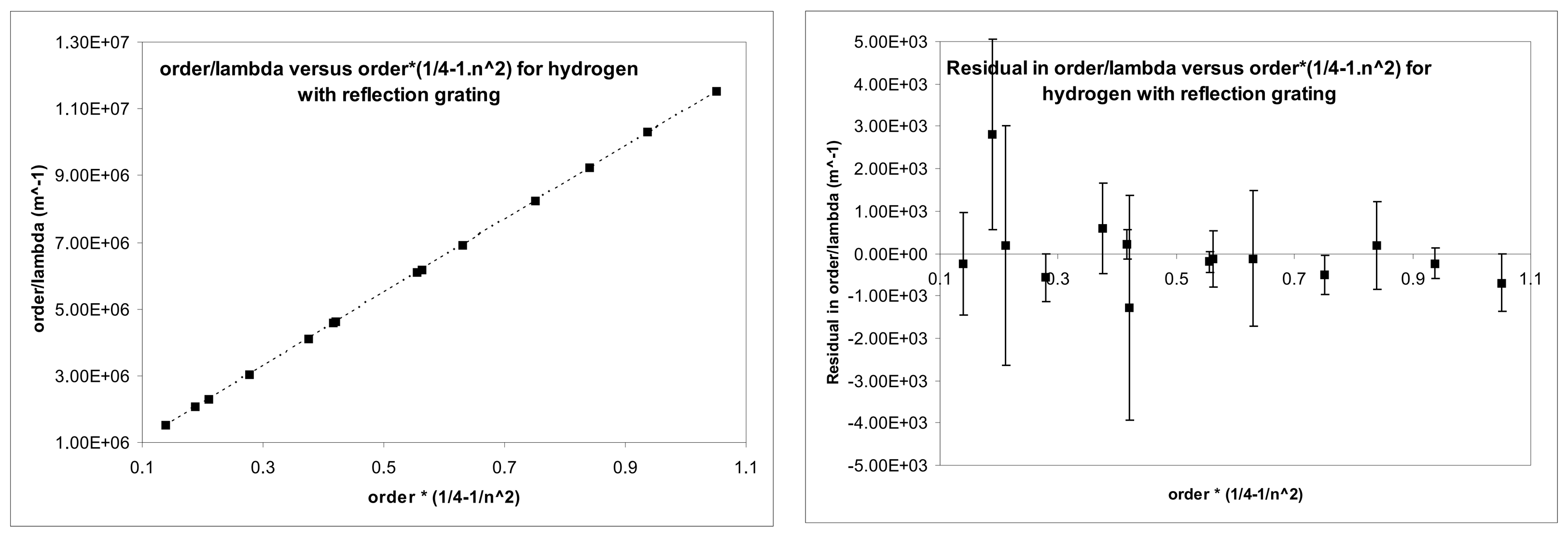

Figure 4: Plot of the measured wavelengths of the H emission lines versus (1/4–1/n^2) for the reflection grating. To separate the different orders of reflections, both *x* and *y* coordinates have been multiplied by the order of the emission line. The error bars are smaller than the symbols used to plot the data. The right-hand graph shows a plot of the residuals from the fit.

Reflection Grating Calibration: Using the same mercury discharge tube as for the transmission grating, the positions of 19 Hg emission lines were measured up to 5th order (Figure 3). Using the known wavelengths of the mercury lines, the line density of the grating was determined to be 620.1(1)lines/mm.

Rydberg Constant of Hydrogen: Using the calibrated transmission grating, the positions of the violet, green and red Balmer lines from a hydrogen discharge tube were measured up to 5th order. The angular positions were measured both left and right of the zeroth order position, and, in all cases, the two angles were the same to within 0.02°. After averaging the two angles, the wavelengths of the emission lines in each order were determined as:

order	1	2	3	4	5
violet	433.6(5)	433.8(5)	433.6(3)	433.6(2)	433.73(13)
green	485.9(5)	485.5(3)	485.67(16)	485.76(11)	485.70(8)
red	655.8(5)	655.9(2)	655.58(15)	655.75(11)	not obs

From a least squares fit of 1/ versus (1/4 1/n^2), Figure 4, constraining the fitted line to pass through the origin, the value of *R* for the transmission grating was 1.09819(14) 10^7m ^{1}

Measuring m_p/m_e using a Deuterium Discharge Tube: Because of the different masses of the hydrogen and deuterium nuclei, the Balmer emission lines from deuterium are at slightly shorter wavelengths than those from hydrogen. The wavelength difference is related to m_p/m_e by:

$$\frac{D}{H} \quad \frac{R_H}{R_D} \quad \frac{1 \quad m_e/2m_p}{1 \quad m_e/m_p} \quad 1 \quad m_e/2m_p$$

Hence

$$\frac{D \quad H}{H} \quad \frac{\ }{H} \quad m_e/2m_p$$

The angular splitting for 4 Balmer lines from a hydrogen/deuterium tube were measured in third, fourth and fifth order. Each line was measured both left and right of the zeroth order position, and, in all cases, the two angles were the same to within 0.01°.

The wavelengths of the 4 lines, and their wavelength splittings ,were:

		H	D	
red	(3rd order)	655.69(11)nm	655.40(15)nm	–0.29(19)nm
red	(4th order)	655.74(11)nm	655.50(11)nm	–0.24(15)nm
green	(4th order)	485.68(11)nm	485.46(11)nm	–0.22(16)nm
green	(5th order)	485.73(8)nm	485.54(8)nm	–0.19(12)nm

The resulting mean value of m_p/m_e is 1200(400), within 1.5 esd's of the literature value of 1836.