Segunda lista de exercícios

Faça estes exercícios do livro do Arfken:

6.1.3 Prove algebraically that for complex numbers,

$$|z_1| - |z_2| \leq |z_1 + z_2| \leq |z_1| + |z_2|.$$

Interpret this result in terms of two-dimensional vectors. Prove that

$$|z-1| < \left|\sqrt{z^2-1}\right| < |z+1|, \qquad \text{for} \quad \Re(z) > 0.$$

6.1.10 Using the identities

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}, \qquad \sin z = \frac{e^{iz} - e^{-iz}}{2i},$$

established from comparison of power series, show that

(a) $\sin(x+iy) = \sin x \cosh y + i \cos x \sinh y,$

$\cos(x+iy) = \cos x \cosh y - i \sin x \sinh y,$

(b) $|\sin z|^2 = \sin^2 x + \sinh^2 y, \qquad |\cos z|^2 = \cos^2 x + \sinh^2 y.$

This demonstrates that we may have $|\sin z|, |\cos z| > 1$ in the complex plane.

6.1.11 From the identities in Exercises 6.1.9 and 6.1.10 show that

(a) $\sinh(x + iy) = \sinh x \cos y + i \cosh x \sin y,$

$\cosh(x + iy) = \cosh x \cos y + i \sinh x \sin y,$

(b) $|\sinh z|^2 = \sinh^2 x + \sin^2 y, \qquad |\cosh z|^2 = \cosh^2 x + \sin^2 y.$

6.1.12 Prove that

(a) $|\sin z| \geq |\sin x| \qquad$ (b) $|\cos z| \geq |\cos x|$.

6.1.13 Show that the exponential function e^z is periodic with a pure imaginary period of $2\pi i$.

6.1.14 Show that

(a) $\tanh \dfrac{z}{2} = \dfrac{\sinh x + i \sin y}{\cosh x + \cos y}, \qquad$ (b) $\coth \dfrac{z}{2} = \dfrac{\sinh x - i \sin y}{\cosh x - \cos y}.$

6.1.15 Find all the zeros of

(a) $\sin z$, (b) $\cos z$, (c) $\sinh z$, (d) $\cosh z$.

6.1.16 Show that

(a) $\sin^{-1} z = -i \ln\left(iz \pm \sqrt{1 - z^2}\right),$ (d) $\sinh^{-1} z = \ln\left(z + \sqrt{z^2 + 1}\right),$

(b) $\cos^{-1} z = -i \ln\left(z \pm \sqrt{z^2 - 1}\right),$ (e) $\cosh^{-1} z = \ln\left(z + \sqrt{z^2 - 1}\right),$

(c) $\tan^{-1} z = \dfrac{i}{2} \ln\left(\dfrac{i + z}{i - z}\right),$ (f) $\tanh^{-1} z = \dfrac{1}{2} \ln\left(\dfrac{1 + z}{1 - z}\right).$

Hint. 1. Express the trigonometric and hyperbolic functions in terms of exponentials. 2. Solve for the exponential and then for the exponent.