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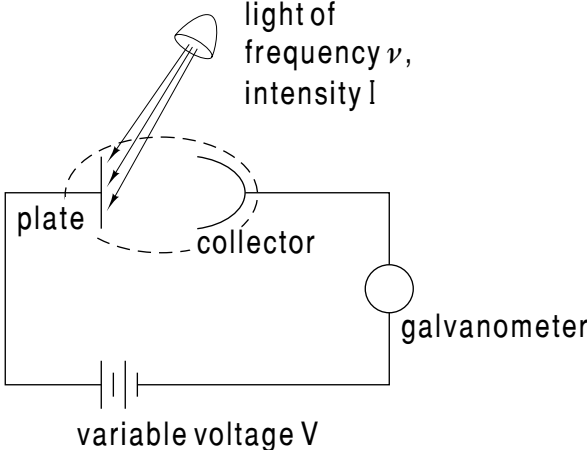
MISN-0-213

THE PHOTOELECTRIC EFFECT

light of frequency ν, intensity I

plate

collector

galvanometer

variable voltage V

THE PHOTOELECTRIC EFFECT
by
M. Brandl

Title: **The Photoelectric Effect**

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Input Skills:

1. Vocabulary: photons (MISN-0-212), electromagnetic wave (MISN-0-210).
2. Calculate the change in potential energy experienced by an electron that falls through a given potential difference (MISN-0-117).

Output Skills (Knowledge):

K1. Vocabulary: cutoff frequency, impeding potential, photoelectric effect, photocurrent, photoelectron, photoemission, stopping potential, work function.

K2. Describe the three aspects of the photoelectric effect which contradict the predictions of the electromagnetic wave theory of light.

K3. Explain how the photon theory of light correctly predicts the experimental observations of the photoelectric effect.

Output Skills (Rule Application):

R1. Given the work function, the density, and the spacing between atoms of a material, and the intensity of light incident on the material's surface, determine how long the electromagnetic wave theory would predict it should take before photoemission can occur.

Output Skills (Problem Solving):

S1. Given two of the following find the third using Einstein's equation for the photoelectric effect: frequency or wavelength of incident light, maximum photoelectron kinetic energy or stopping potential, work function of metal or cutoff frequency.

Post-Options:

1. "The Compton Effect" (MISN-0-219).
2. "Wave-Particle Duality: Light" (MISN-0-246).

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THE PHOTOELECTRIC EFFECT

by

M. Brandl

1. Overview

1a. Macroscopic View of Light. On the macroscopic, everyday-world scale, light clearly exhibits the properties of a wave. It undergoes refraction at boundaries between different media, reflects at surfaces, diffracts around obstacles, and exhibits interference: all of these are wave phenomena. The wave nature of light was given a solid theoretical basis in 1864 by Maxwell's prediction of the existence of electromagnetic waves produced by oscillating electric charges.[1]

1b. Microscopic View of Light. On the microscopic scale, however, the electromagnetic wave picture of the nature of light breaks down. Molecules, atoms, nuclei, and elementary particles do not "see" light as an electromagnetic wave; that is, they do not interact with light as they should interact with a simple set of time-varying electric and magnetic fields. Rather, they seem to interact as if a beam of light were made up of discrete particles, each one carrying a specific amount of energy, momentum, and angular momentum. These particles are called photons.[2]

1c. Photoemission Supports the Photon Model. The photoelectric effect produced the first conclusive experimental evidence for the photon picture of light. It demonstrates that atomic electrons absorb energy from a beam of electromagnetic radiation in discrete amounts, which shows that a beam of electromagnetic radiation is made up a small discrete "bundles" of energy.

2. The Experimental Evidence

2a. First Observations. The photoelectric effect was first noted by Heinrich Hertz in 1887. He observed that if an electric discharge between electrodes is illuminated with ultraviolet light, the intensity of the discharge is increased. This implies that more electrons are freed from the electrodes to jump across the gap. In 1888, Wilhelm Hullwachs observed that electrons are emitted from the surfaces of such metals as zinc, rubid-

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[1]See "Electromagnetic Waves and Maxwell's Equations" (MISN-0-210).
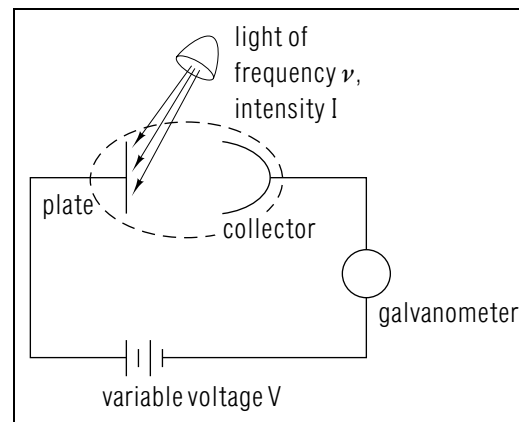[2]See "Characteristics of Photons" (MISN-0-212).

Figure 1. Millikan's photoelectric effect experiment.

ium, phosphorus, and sodium when they are struck by a beam of light. This effect was called "photoemission." Obviously, some electrons in the metals are able to absorb enough energy from a beam of light to escape from the surfaces of the metals. This particular observation in itself can be explained by the electromagnetic wave picture of light: the electrons are accelerated by the electric field in the wave until they gain enough kinetic energy to break out of the metal. Experiments were then devised to examine the details of the process of photoemission, and these details showed the electromagnetic wave picture of light to be untenable on a microscopic scale.

2b. Millikan's Photoelectric Experiment. The definitive experimental work on the photoelectric effect was done by R. A. Millikan in 1914 (for which he was awarded a Nobel prize in 1923). The experimental setup is shown schematically in Fig. 1. A vacuum tube is composed of a metal plate and a metal collecting cup. The plate is illuminated with light of known intensity I and known frequency ν. Most of the electrons freed from the plate by the light, called "photoelectrons," travel to the collector. A variable voltage supply producing a voltage V is connected across the plate and collector. If $V < 0$, the collector is positively charged relative to the plate, and the photoelectrons are accelerated as they cross the tube. If $V > 0$, the collector is negatively charged relative to the plate, and the progress of the photoelectrons to the collector is impeded. In this case the potential difference is called an "impeding potential." Since the electrons are ejected from the plate with some initial kinetic energy, some of them will make it to the collector despite the electric field that is trying to turn them back. Any electron having an initial kinetic
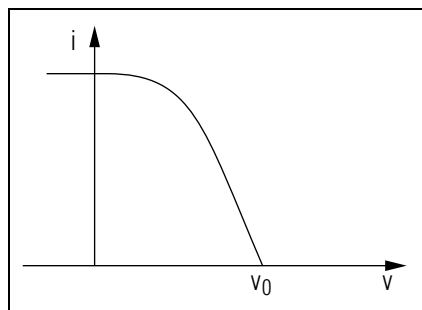
Figure 2. Plot of photoelectric current against impeding potential.

energy greater than the potential energy difference between the plates will still strike the collector. Those photoelectrons which do arrive at the collector return to the plate through the circuit. A galvanometer measures the "photocurrent," i, flowing in the circuit, which is proportional to the number of photoelectrons striking the collector per unit time. If an impeding potential V is placed across the plate and collector, the galvanometer reading indicates how many photoelectrons per unit time are liberated from the plate with an initial kinetic energy greater than the potential energy difference between the plates.

2c. Results of Millikan's Experiment. The experiment itself is performed by setting the intensity I and frequency ν of the light illuminating the plate to known values, and then measuring the photocurrent as a function of the impeding potential. The results look like those shown in the Fig. 2. When $V < 0$, nearly all of the photoelectrons reach the collector, so the photocurrent is large.[3] As V become positive, the external electric field opposes the motion of the photoelectrons, so the least energetic photoelectrons are kept from striking the collector, and the photocurrent drops. As V is increased, fewer and fewer photoelectrons can reach the collector. Finally, at a critical potential called the "stopping potential," V_0, none of the electrons have an initial kinetic energy high enough to reach the collector, and the photocurrent drops to zero. If V is increased above V_0, the photocurrent remains at zero.

This implies that, for a given intensity and frequency of light striking the plate, there exists a maximum value for the kinetic energy of the

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[3]The photocurrent is actually larger for more negative values of V because the externally applied electric field helps free more electrons from the plate, but this effect is irrelevant to the purpose of the experiment.
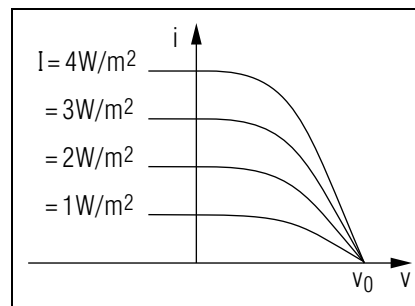
Figure 3. Dependence of photocurrent on light intensity.

photoelectrons when they are emitted from the plate given by

$$E_{k,max} = eV_0 \,. \tag{1}$$

Not all of the electrons are liberated from the plate with this kinetic energy; most have less. This can be explained in terms of the way in which the electrons are bound to the metal of the plate.[4] The kinetic energy of each photoelectron is determined by the energy absorbed by each photoelectron from the light, E_L, minus the energy required to leave the metal surface, ϕ:

$$E_K = E_L - \phi \,. \tag{2}$$

The maximum photoelectron kinetic energy occurs for photoelectrons with the minimum binding energy, ϕ_0, called the "work function" of the metal surface.

$$E_{K,max} = E_L - \phi_0 = eV_0 \,. \tag{3}$$

By examining the characteristics of the stopping potential V_0 or the maximum kinetic energy $E_{K,max}$ we are also examining the characteristics of E_L, the amount of energy absorbed from the incident light by each electron. We will now see what happens to E_K and the photocurrent when the intensity I and frequency ν of incident light are varied, and we shall also see how the photocurrent varies as a function of the amount of time the light has been on.

2d. Dependence on Intensity of Incident Light. Figure 3 is a plot of photocurrent versus impeding potential for several different values of the incident light intensity (with the frequency being held constant). The current is larger when the intensity is larger, which means that the more intense the light is, the more photoelectrons are freed from the

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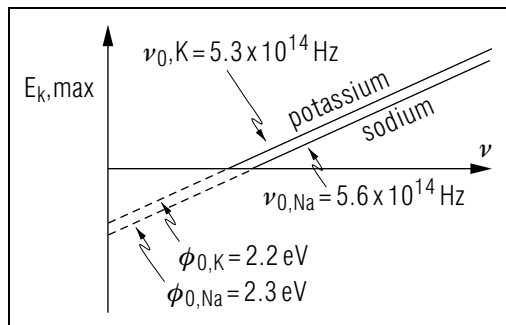[4]See "Appendix A: Explanation of the Work Function."

Figure 4. Maximum photoelectron kinetic energy as a function of frequency.

metal.[5] However, the value of the stopping potential is the same for all values of the intensity. Changing the intensity of the light striking the plate changes only the number of photoelectrons emitted. There is no measurable minimum value of the intensity required for photoemission to take place. This implies that the amount of energy absorbed from the light by each electron is independent of the intensity of the light. This result has been tested and found valid over an intensity range of seven orders of magnitude.

2e. Dependence on Frequency of Incident Light. Figure 4 is a plot of the maximum photoelectron kinetic energy as a function of the frequency of the light illuminating the plate. The results of two different experiments are shown, one using a potassium plate and the other using a sodium plate. In each case the resulting graph is a straight line, indicating that the maximum kinetic energy depends linearly on the frequency. The slopes of the lines are the same for both metals, and would be the same for any metal used to form the plate. The value of the slope, according to Millikan's original results, is $(6.57 \pm 0.33) \times 10^{-34}$ J s. Later experiments gave a more precise value of 6.62517×10^{-34} J s, which is the value of Planck's constant. For each metal, there is a definite "cutoff frequency," ν_0, below which no photoelectrons are emitted, regardless of the intensity of the light striking the plate. The value of the cutoff frequency is $\nu_0 = \phi_0/h$, where ϕ_0 is the work function of the metal of which the plate is constructed. The equation of the line for each metal is, therefore,

$$E_{K,max} = h(\nu - \nu_0) = h\nu - \phi_0 \, ,$$

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[5]The photocurrent is, in fact, directly proportional to the intensity of the incident light at a fixed value of the impeding potential.

which implies that the amount of energy absorbed by each electron is

$$E_L = h\nu. \tag{4}$$

The amount of energy absorbed by each electron from the light beam depends solely on the frequency of the light. When the frequency of the light striking the plate is less than ν_0, each electron absorbs an amount of energy $E_L \le \phi_0$, which is not enough to free it from the plate. For such frequencies no photoelectrons are emitted.

2f. Time Dependence of Photoemission. Attempts have been made to determine how long it takes for the electrons to absorb enough energy from the light to escape from the plate. No detectable time lag has been found. The electrons absorb the energy from the light beam nearly instantaneously.[6] This result is independent of the intensity and frequency of the incident light (so long as the frequency is high enough to cause photoemission, of course).

3. The Classical Interpretation

3a. Description of the Mechanism. In the electromagnetic wave picture of the photoelectric effect, the beam of light which is incident on the plate is simply a region of space in which there exists time-varying electric and magnetic fields. The electrons in the plate are accelerated by the electric field of the wave and oscillate harmonically in phase with the field. They pick up more energy with each oscillation until they absorb enough energy to escape from the plate. They are further accelerated by the field as they leave the plate, and thus acquire some additional kinetic energy.

3b. Dependence on Intensity of Incident Light. The electromagnetic wave interpretation predicts that the amount of energy absorbed from the incident light by each electron depends on the intensity of the light. The intensity of an electromagnetic wave is proportional to the square of the amplitude of the electric field of the wave. Thus if the intensity of the incident light is increased, the amplitude of the electric field in the light is also increased. This in turn means that the electric force accelerating the electrons in the plate is greater, so the electrons absorb more energy from the light. This would make the stopping potential depend on the intensity, a conclusion which contradicts the experimental

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[6]An upper limit of 10^{-9} sec has been set on the time lag. Smaller time intervals can not be measured.

results.

3c. Dependence on Frequency of Incident Light. The electromagnetic wave model of the photoelectric effect does not predict the linear dependence of the maximum photoelectron kinetic energy on the frequency of the incident light. Furthermore, it predicts that photoemission will occur for any frequency of the incident light, as long as the intensity is high enough. This prediction contradicts the experimental observation of the existence of a cutoff frequency, below which no photoemission occurs regardless of how high the intensity is.

3d. Time Dependence of Photoemission. The electromagnetic wave interpretation of the photoelectric effect predicts that there will be a finite, measurable delay between the time when light begins to strike the surface of the metal and the emission of the first photoelectron. We can easily derive a classical expression for the approximate lower limit of this time:[7]

$$t_{em} = \frac{\phi_0}{I}\left(\frac{\rho}{\mu}\right)^{2/3},$$

where ϕ, ρ, and μ are the work function, density, and mass-per-atom of the metal and I is the intensity of the light. As an example, potassium has a work function of $1.8\,eV = 2.9 \times 10^{-19}$ J, a density of $860\,\mathrm{kg/m^3}$, and each atom has a mass of approximately 6.5×10^{-26} kg. If we now allow light whose intensity is the same as that of sunlight, $10^3\,\mathrm{W/m^2}$, to illuminate a piece of potassium, the time it will take before the first photoelectron is emitted is

$$t_{em} = \frac{2.9 \times 10^{-19}\,\mathrm{J}}{\left(10^3\,\mathrm{W/m^2}\right)\left(4.2 \times 10^{-10}\,\mathrm{m}\right)^2} = 1.6 \times 10^{-3}\,\mathrm{s}.$$

(Each atom is in a cube of side 4.2×10^{-10} m.)

Such a time lag would be easily detectable. For lower intensities, the time lags should be even longer. For a more extreme case, an intensity of $10^{-12}\,\mathrm{W/m^2}$ would yield a time lag of 1.6×10^4 s which is 4 hours and 34 minutes!

The prediction of such a time lag definitely contradicts the experimental observation that photoemission occurs nearly instantaneously.

3e. Failure of the Electromagnetic Wave Model. As the previous three paragraphs show, the electromagnetic wave picture of light

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[7]See "Appendix B: Time Delay for 'Classical' Photoemission."

completely fails to explain the details of the photoelectric effect. It is still true that light acts like an electromagnetic wave on the macroscopic scale, that is, a beam of light still behaves exactly like a region of space filled with a continuous, time-varying electric and magnetic field when we observe its behavior on a scale of distances much larger than the size of an atom. But we must now revise our picture of the way light interacts with matter on a microscopic scale.

4. The Photon Model

4a. Planck's Quantum Hypothesis. The photon theory of electromagnetic radiation was first proposed by Albert Einstein in 1905 as a result of Max Planck's theoretical work on blackbody radiation. Planck found that in order to explain the way in which radiation is emitted from a heated object, it was necessary to assume that it was emitted in discrete "bunches," each bunch containing an amount of energy $h\nu$, where ν is the frequency of the emitted radiation and h is Planck's constant. Planck assumed that this "quantization" was solely a property of the process by which the radiation was emitted, and that the total outgoing stream of radiation could still be considered a continuous electromagnetic wave.

4b. Einstein's Photon Postulate. Einstein carried Planck's quantization idea one step further and postulated that electromagnetic radiation itself was quantized—that a beam of electromagnetic radiation consists of numerous concentrated bundles, called photons. The energy carried by a single photon is

$$E_{ph} = h\nu, \tag{5}$$

where ν is the frequency of the radiation and h is Planck's constant. Not only is electromagnetic radiation emitted in discrete bundles, but it retains that same "granular" structure as it propagates through space. It is therefore also absorbed in discrete amounts.

4c. Macroscopic Beams Contain Many Photons. The amount of energy contained in each photon is so small that a beam of electromagnetic radiation must contain a very large number of photons in order for it to be detectable on a macroscopic scale. Therefore, it is impossible to detect the granularity of radiation on a macroscopic scale. All we can see is the net effect of a large number of photons, which causes a beam to appear to be a continuous electromagnetic wave. The situation is analogous to the structure of matter: on the microscopic level, a table is a complex assortment of molecules (or, on an even smaller scale, of

elementary particles), but on a macroscopic scale it looks and acts like a continuous, solid piece of material. Following the same analogy, just as we need not worry about the microscopic structure of a table in using it in the everyday world, we also have no need to worry about the microscopic structure of light when we deal with its macroscopic properties. In the everyday world, a table is a solid piece of material, and a beam of light is an electromagnetic wave.

4d. Microscopic Systems Interact with Photons. It is only on the microscopic level of molecules, atoms, nuclei, and elementary particles that the quantized nature of electromagnetic radiation manifests itself. On this level, a beam of electromagnetic radiation of frequency ν consists of a stream of photons, each one having energy $h\nu$. The interaction between electromagnetic radiation and matter on a microscopic scale can therefore be described in terms of the interactions between individual particles and individual photons. In such interactions the photons display particle-like properties, with each one having a definite energy and momentum.[8]

5. The Photon Interpretation

5a. Introduction. In his 1905 paper, Einstein pointed out that the photoelectric effect provided an excellent experimental test of his photon theory. His predictions were borne out by the experiments, and he received a Nobel prize in 1921 for his work.

5b. Einstein's Photoelectric Equation. The photon interpretation of the photoelectric effect is particularly simple. When a beam of light strikes the surface of a metal, some of the photons in the beam strike some of the electrons in the metal. Each electron involved absorbs a single photon, so the net amount of energy absorbed by one electron from a beam of light of frequency ν is the amount contained in a single photon:

$$E_L = E_{ph} = h\nu.$$

An electron bound in the metal by an energy ϕ would then escape with kinetic energy

$$E_k = h\nu - \phi.$$

This equation is usually called Einstein's equation for the photoelectric effect.

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[8]In fact, photons act precisely like particles having zero rest mass; the particle dynamics involved are examined in "The Compton Effect" (MISN-0-219).

5c. Dependence on Intensity of Incident Light. The photon theory predicts that the photoelectron kinetic energy is independent of the incident light intensity, and that the photoelectric current is proportional to the intensity. Both predictions are borne out by experiment. In the photon picture of electromagnetic radiation, the energy carried by each photon in a beam of frequency ν is $h\nu$, which is a fixed amount. The intensity is therefore a measure of the number of photons in the beam:

$$I = h\nu\phi_{ph},$$

where ϕ_{ph} is called the "photon flux," number of photons striking a surface of unit area perpendicular to the beam per unit time. A more intense beam just has more photons in it: the energy of each photon is independent of the intensity. Therefore, since each electron absorbs just a single photon, the amount of energy absorbed from the light by each electron is independent of the intensity of the light. Thus the maximum photoelectron kinetic energy (and hence the stopping potential) is independent of the light intensity. This prediction is completely in accord with the experimental evidence. Changing the intensity of the light changes the number of photons striking the plate, and so presumably changes the number of electrons which absorb photons. One would therefore expect that the number of photoelectrons emitted per unit time (the photoelectric current) would be proportional to the intensity of the light. This is also in agreement with the experimental results.

5d. Dependence on Frequency of Incident Light. The photon theory makes the correct prediction about the relationship between the maximum photoelectron kinetic energy E_k, max and the frequency ν of the incident light:

$$E_{k,max} = h\nu - \phi_0,$$

where h is Planck's constant and ϕ_0 is the work function of the metal. The particular prediction is verified by the experimental results. The slope of the graph of $E_{k,max}$ vs. ν is indeed equal to Planck's constant, and the intercept is equal to the work function, within the limits of experimental error. The photon theory also correctly predicts the existence of the cutoff frequency, below which no photoemission occurs. If the frequency ν of the incident light is less than ϕ_0/h, each photon has an energy of $h\nu/\phi_0$. Therefore since the electrons are all bound by an amount $\phi \geq \phi_0$, no electron can absorb enough energy to escape from the metal by "swallowing" a single photon.[9] No photoemission occurs for a frequency

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[9]An electron could conceivably escape by absorbing two photons sequentially, but the chances of this occurring are vanishingly small.

less than $\nu_0 = \phi_0/h$.

5e. Time Dependence of Photoemission. The photon interpretation of the photoelectric effect predicts that there will be no appreciable time lag between light striking the surface of a metal and the start of photoemission. This is in complete agreement with the experimental results. Each electron absorbs the energy it needs to escape in a finite but very small interval of time. We can make a rough guess at the value of the amount of time the absorption occurs in, by setting it equal to the length of time it would take for a photon (traveling at the speed of light) to cross an electron. The size of an electron is not a well-determined quantity, but is of the order of 10^{-14} m. Therefore, the absorption time is, roughly,

$$t_{abs} = \frac{10^{-14}\,\text{m}}{3 \times 10^8\,\text{m/s}} = \frac{1}{3} \times 10^{-22}\,\text{s} \,.$$

There is no way in which such a short time interval could be measured.

5f. Success of the Photon Interpretation. The photon theory of light successfully explains all of the observed details of the photoelectric effect. We must therefore conclude that, on a microscopic level, electromagnetic radiation is quantized, and that its interaction with matter on this level must be described in terms of the interactions of individual photons with individual particles.

Acknowledgments

Glossary

- **cutoff frequency**: the lowest frequency of light that will cause the emission of electrons from a metal surface.

- **impeding potential**: the potential difference, between a metal plate and a collector, that decelerates photoelectrons emitted from the metal plate.

- **photocurrent**: the current of electrons between the metal plate and collecting cup in a vacuum tube illuminated by light of sufficiently high
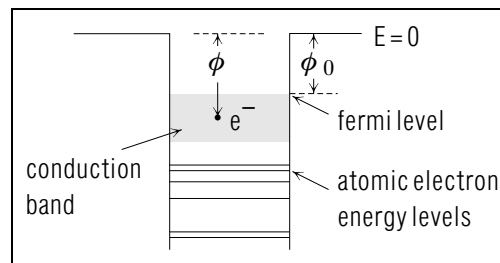
Figure 5. Electrons in a metal.

frequency.

- **photoelectric effect**: the emission of electrons from a metal surface exposed to light of sufficiently high frequency.

- **photoelectrons**: the name given to electrons emitted from a metal surface via the photoelectric effect.

- **photoemission**: another name for the photoelectric effect.

- **stopping potential**: the impeding potential required to completely retard the photocurrent.

- **work function**: the minimum energy binding electrons to a metal surface.

A. Explanation of the Work Function

The conduction electrons in a metal are not bound to individual atoms of the crystal lattice, but are free to move about inside the metal. They are, however, bound to the metal itself by a potential barrier at the surface. In order to escape from the metal, these electrons must be given enough energy to overcome this barrier. The electrons in a metal are therefore sitting in a potential well. Each electron's "depth," ϕ, in the well is the amount by which it is bound to the metal (see Fig. 5).[10] Different electrons are situated at different depth in the well, since the Pauli exclusion principle requires them all to be in different states.

Each electron struck by a photon absorbs the energy of the photon. The dynamics of the collision process are rather complicated. Since both

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[10]That is, an amount of work equal to ϕ must be done to remove the electron from the metal.

momentum and energy must be conserved, a free electron cannot absorb all of the energy of a photon because there would be some "excess" momentum left over.[11] However, an electron bound inside a metal is able to transfer this excess momentum to the crystal structure of the metal, and escape with all of the photon's energy.

An electron bound at depth ϕ must be given an amount of energy equal to ϕ to enable it to escape from the metal. Any additional energy given to it will appear as kinetic energy after it is liberated. Therefore, if an electron bound at depth ϕ absorbs a total amount of energy E_L from light striking the metal, its kinetic energy upon leaving the metal will be

$$E_k = E_L - \phi. \tag{6}$$

Of course, if $E_L < \phi$, the electron will not be emitted.

Different electrons are bound at different "depths" ϕ. Assuming each electron absorbs the same amount of energy from the incident light, those which are least tightly bound will be liberated with the greatest kinetic energy.

There exists a minimum value of the binding energy ϕ, for electrons at the very top of the conduction band (at the Fermi level). This value, called ϕ_0, the work function of the metal, is solely a property of the metal involved. Therefore, if we assume that all the electrons absorb the same amount of energy E_L from the incident light, the maximum kinetic energy with which an electron can be emitted from the metal is

$$E_{k,max} = E_L - \phi_0 = eV_0. \tag{7}$$

B. Time Delay for "Classical" Photoemission

We can use a simple argument to set an approximate lower limit on the amount of time it should take for an electron at the surface of a metal to absorb enough energy from an electromagnetic wave to escape from the metal.

Assume that there is one free electron per atom in the metal. If we also assume that the electrons on the surface of the metal are evenly distributed, then each electron can be imagined as occupying a small

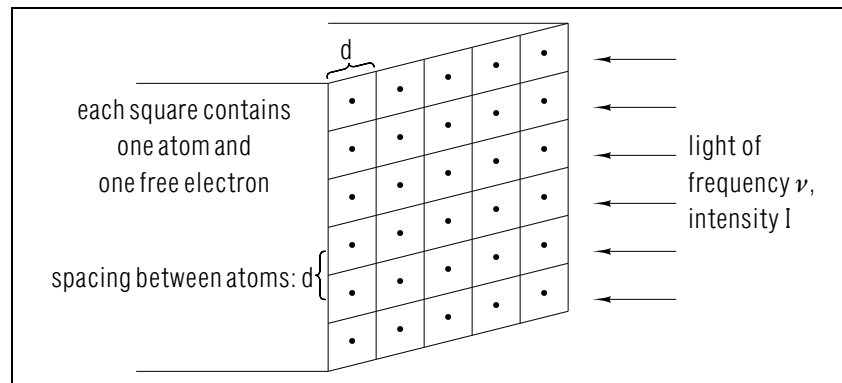[11] See "The Compton Effect" (MISN-0-219).

Figure 6. energy absorption from incident light.

square about an atom on the surface. The size of this square is determined by the spacing d of the atoms (see Fig. 6).

Each electron can absorb the energy that falls on its own square. The maximum rate at which an electron can absorb energy from the incident light is simply the total amount of energy falling on its square per unit time,

$$\left(\frac{dE_L}{dt}\right)_{max} = Id^2, \tag{8}$$

where I is the intensity of the light, and d^2 is the area of the electron's square.

The maximum amount of energy an electron can absorb from the light in time t is, therefore,

$$E_{L,max} = Id^2t. \tag{9}$$

The value of d can be estimated from the mass density of the metal and the mass of each atom, if we assume that the atoms in the crystal are arranged in a simple cubical array. This assumption will not generally be true, and may cause an error of a factor of two or so in the result, but we're only looking for an approximate final answer anyway.

If the atoms are arranged in a simple cubical array with spacing d, then we can divide up the metal into a number of cubes whose sides have length d, with each cube containing one atom. Since the volume of each cube is d^3, there is one atom per each d^3 of volume in the metal. The

number density of atoms is, therefore,

$$N_a = \frac{1}{d^3} . \tag{10}$$

The number density of atoms can also be determined from the mass density ρ of the metal and the mass μ of each atom:

$$N_a = \frac{\rho}{\mu} . \tag{11}$$

Combining the two expressions and solving for d gives

$$d = \left(\frac{\mu}{\rho}\right)^{1/3} . \tag{12}$$

Finally, we can determine how long it should take for photoemission to occur. The time t_{em} required for emission to occur is just the amount of time needed for the electron to absorb an amount of energy equal to the amount by which it is bound to the metal, ϕ. The first electrons emitted will be the ones with the smallest binding energy, that being equal to the work function of the metal, ϕ_0. Therefore, we can find the minimum value of t_{em} by setting $E_{L,max} = \phi_0$ in Eq. (11) and solving for t, which gives

$$t_{em} = \frac{\phi_0}{Id^2} . \tag{13}$$

Combining Eqs. (12) and (13), the time delay is

$$t_{em} = \frac{\phi_0}{I} \left(\frac{\rho}{\mu}\right)^{2/3} . \tag{14}$$

PROBLEM SUPPLEMENT

$$h = 6.62559 \times 10^{-34} \, \mathrm{J\,s} = 4.13557 \times 10^{-15} \, \mathrm{eV\,s}$$
$$\mathrm{J} = 6.24181 \times 10^{18} \, \mathrm{eV}$$
$$c = 3.00 \times 10^8 \, \mathrm{m/s}$$

1. The longest wavelength of light that will cause photoemission from sodium is approximately $540 \, \mathrm{nm}$.

   a. Find the work function of sodium.

   b. Find the maximum kinetic energy for photoelectrons emitted when light of wavelength $400 \, \mathrm{nm}$ strikes a sodium plate.

2. Pure silver has a work function of $\phi_0 = 4.7 \, \mathrm{eV}$. A crude calculation of the type used in the text, using the atomic weight and density of silver, gives a spacing between atoms in a silver crystal of about $d = 12 \, \mathrm{nm}$. Note that d-cubed was taken to be the mass-per-atom divided by the density of the silver. It has been found that light of intensity $1 \times 10^{-10} \, \mathrm{W/m^2}$ can still cause photoemission from silver. If the electromagnetic wave interpretation were correct, how long would it take before the first photoelectrons were emitted?

3. When light of wavelength $244 \, \mathrm{nm}$ illuminates an aluminum surface, a stopping potential of $1.0 \, \mathrm{V}$ is needed to reduce the photoelectric current to zero.

   a. Find the work function of aluminum.

   b. Find the wavelength of light that would require a stopping potential of $1.5 \, \mathrm{V}$.

Brief Answers:

1. a. $\phi_0 = 2.3 \, \mathrm{eV}$.

   b. $K_{E,max} = 0.8 \, \mathrm{eV}$.

2. $t_e m = 5.23 \times 10^7 \, \mathrm{s} = 1.66 \, \mathrm{years}$

3. a. $\phi_0 = 4.08 \, \mathrm{eV}$

   b. $\lambda = 222 \, \mathrm{nm}$.

MODEL EXAM

$$h = 6.62559 \times 10^{-34} \, \text{J s} = 4.13557 \times 10^{-15} \, \text{eV s}$$
$$\text{J} = 6.24181 \times 10^{18} \, \text{eV}$$
$$c = 3.00 \times 10^8 \, \text{m/s}$$

1. See Output Skills K1-K4 in this module's *ID Sheet*.

2. The photoelectric work function of aluminum is $\phi_0 = 4.08 \, \text{eV}$.

   a. Find the cutoff frequency and the corresponding cutoff wavelength for photoemission from aluminum.

   b. Find the maximum photoelectron kinetic energy and the stopping potential when (ultraviolet) light of wavelength 200 nm illuminates an aluminum plate.

3. Photoemission can occur in a gas, as well as in a metal. Consider the (extremely hypothetical) case of a monatomic hydrogen gas with all of the atoms being in their ground state. Each electron is bound to its corresponding nucleus by an amount equal to 13.6 eV. Since the radius of an electron's orbit in a hydrogen atom is $r = 0.053 \, \text{nm}$, each electron can absorb only that amount of energy from a beam of light that falls on a disc whose area is $\pi r^2 = 8.82 \times 10^{-21} \, \text{m}^2$. If light of intensity $I = 1.00 \, \text{W/m}^2$ illuminates the gas, find how long the electromagnetic wave theory would predict it should take for photoemission to occur.

Brief Answers:

1. See this module's *text*.

2. a. $\nu_0 = 9.87 \times 10^{14}/\text{s}$, $\lambda_0 = 304.1 \, \text{nm}$

   b. $E_{k,max} = 2.12 \, \text{eV}$, $V_0 = 2.12 \, \text{V}$

3. $t_{em} = 247 \, \text{s}$.